Quantum Cyber Security
Past - Present - Future

Elham Kashefi

University of Edinburgh
CNRS, Pierre and Marie Curie University

Oxford Quantum Technology Hub
Paris Centre for Quantum Computing

Dec 1969

Dec 1969

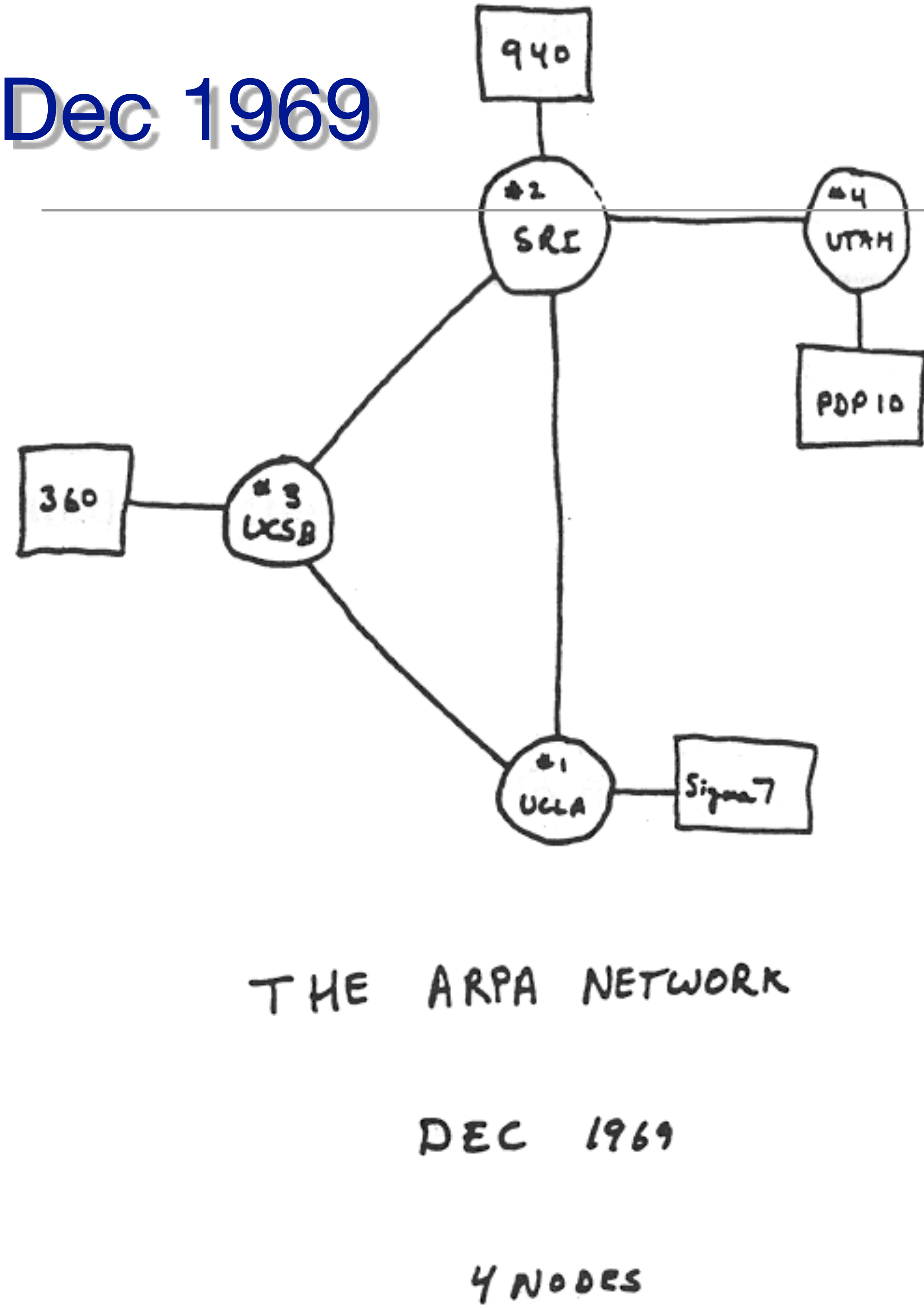
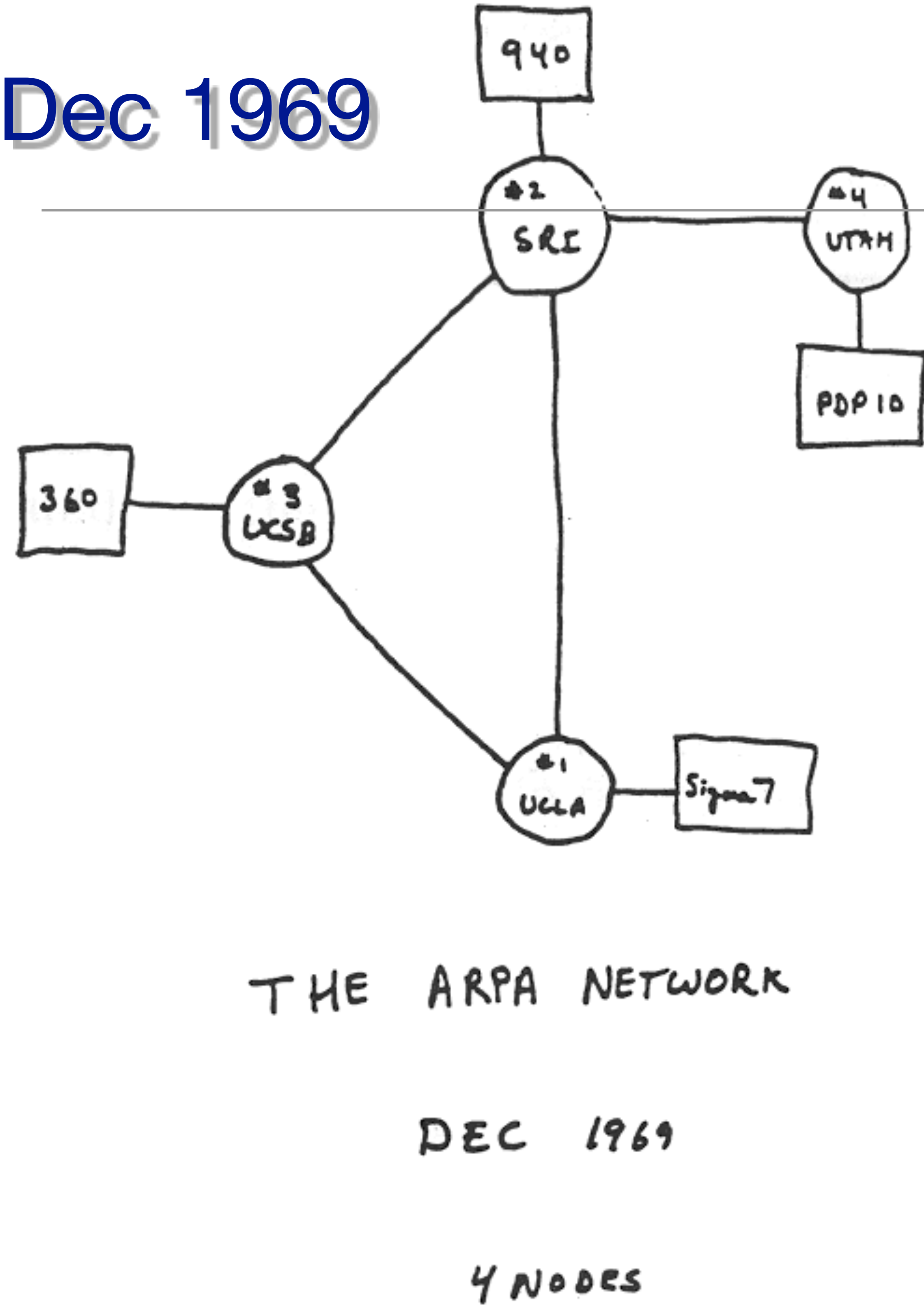


FIGURE 6.2 Drawing of 4 Node Network
(Courtesy of Alex McKenzie)

Dec 1969



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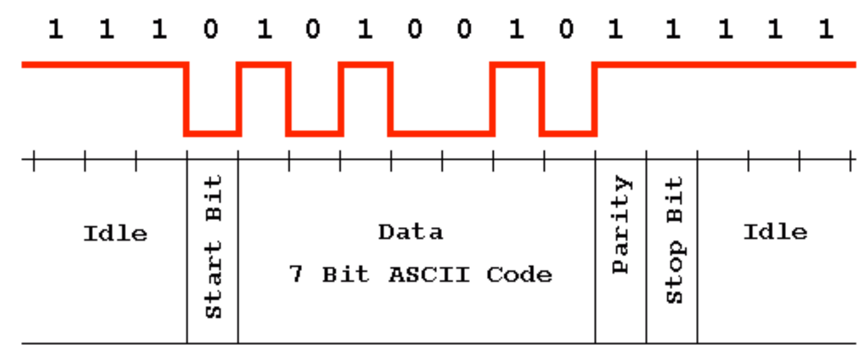
bits communications



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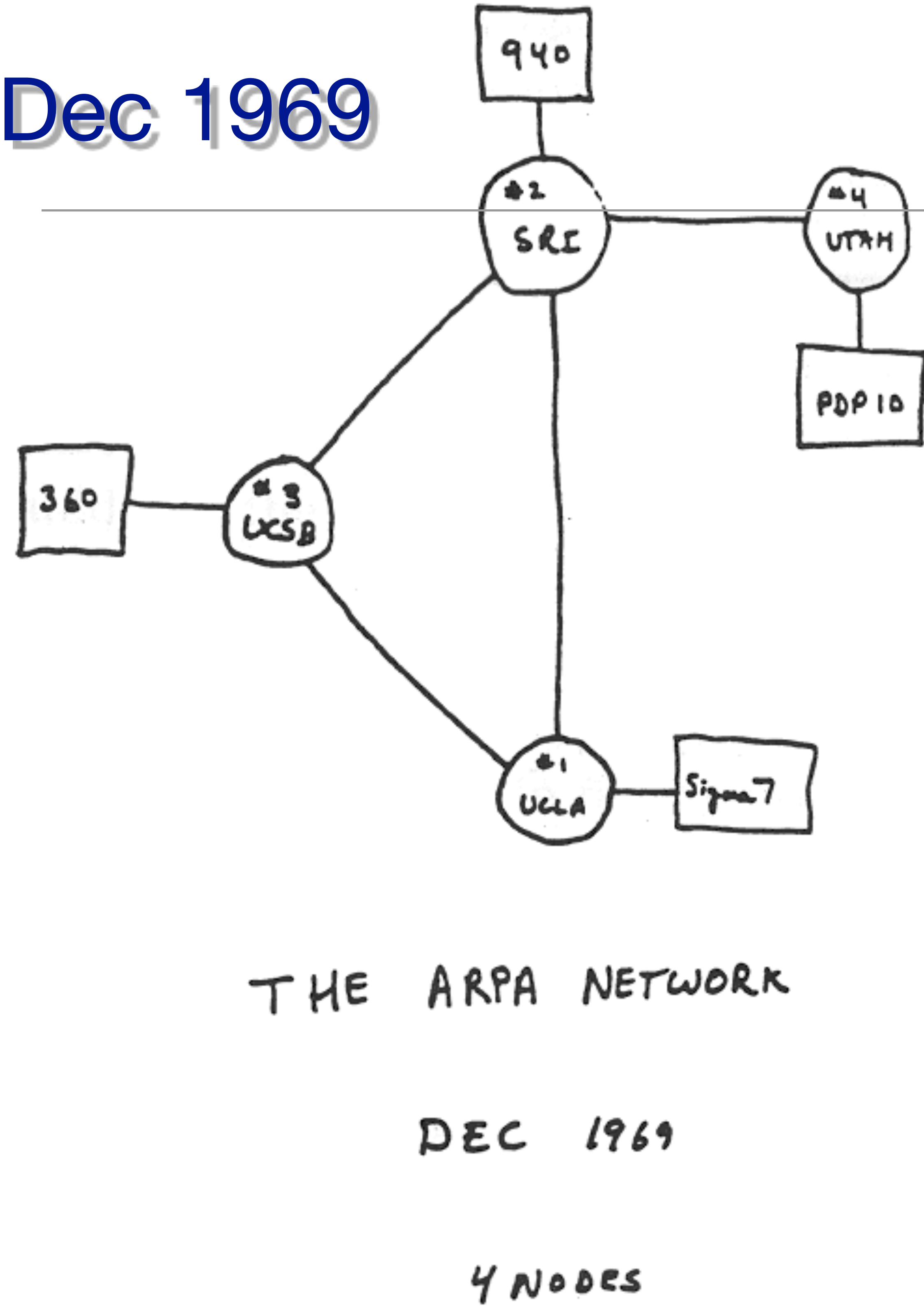

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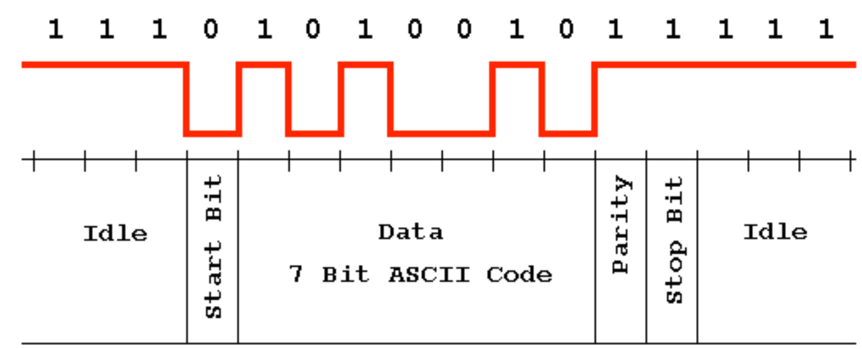

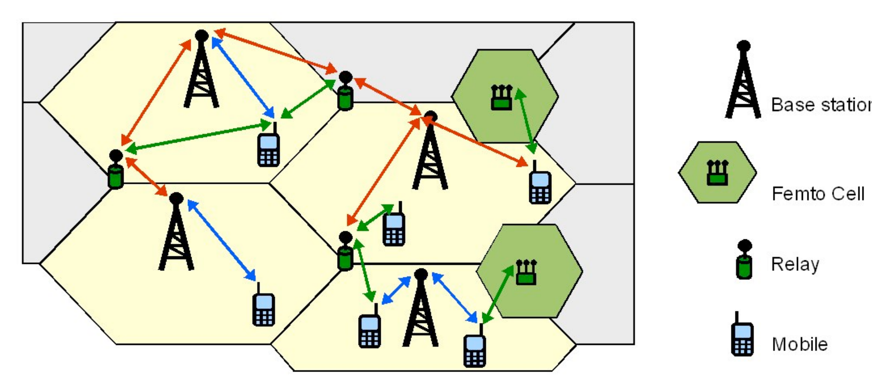

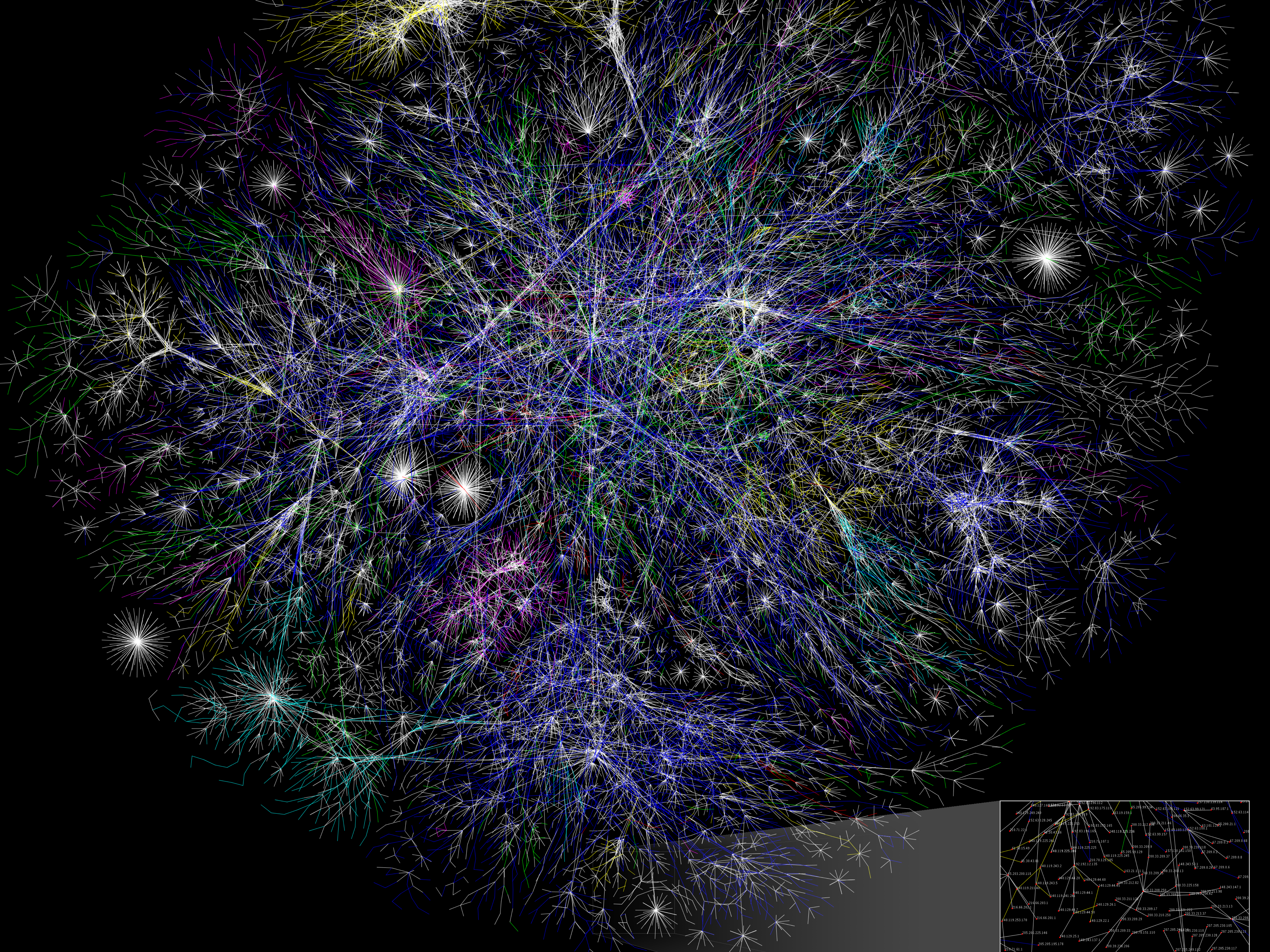

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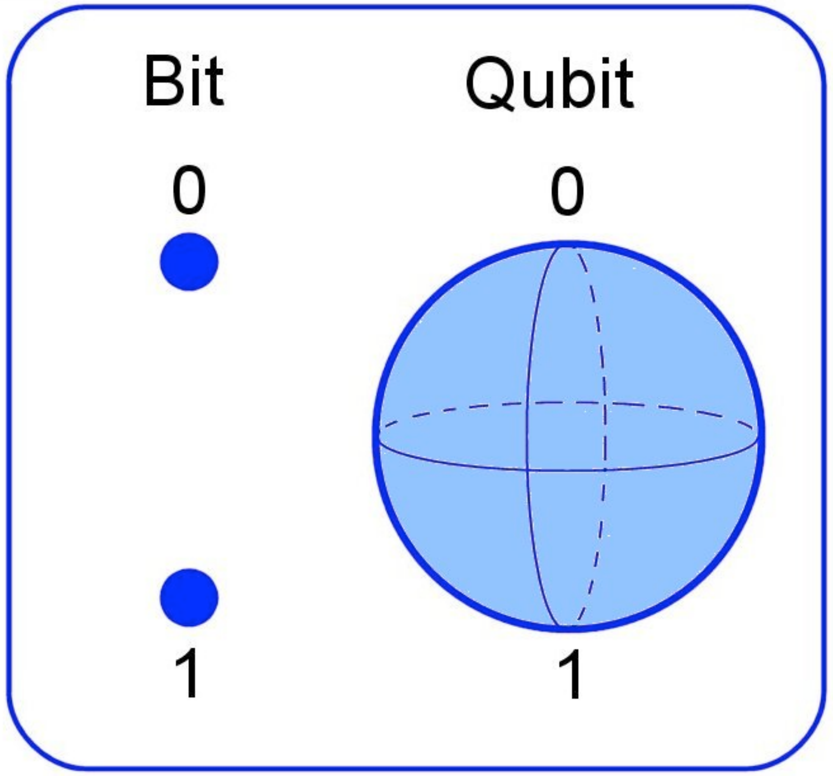

single qubits communications

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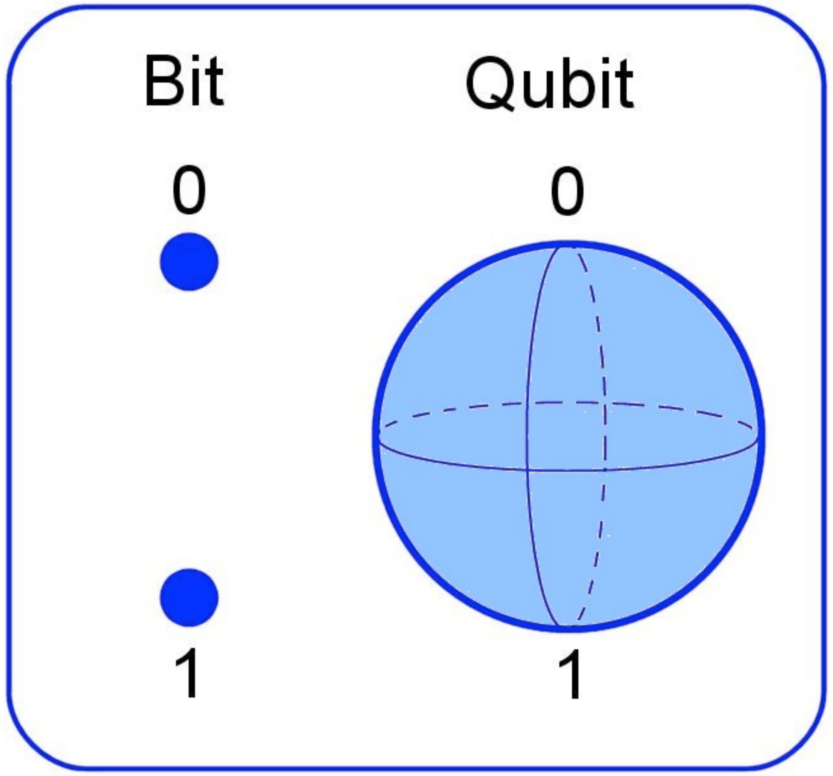

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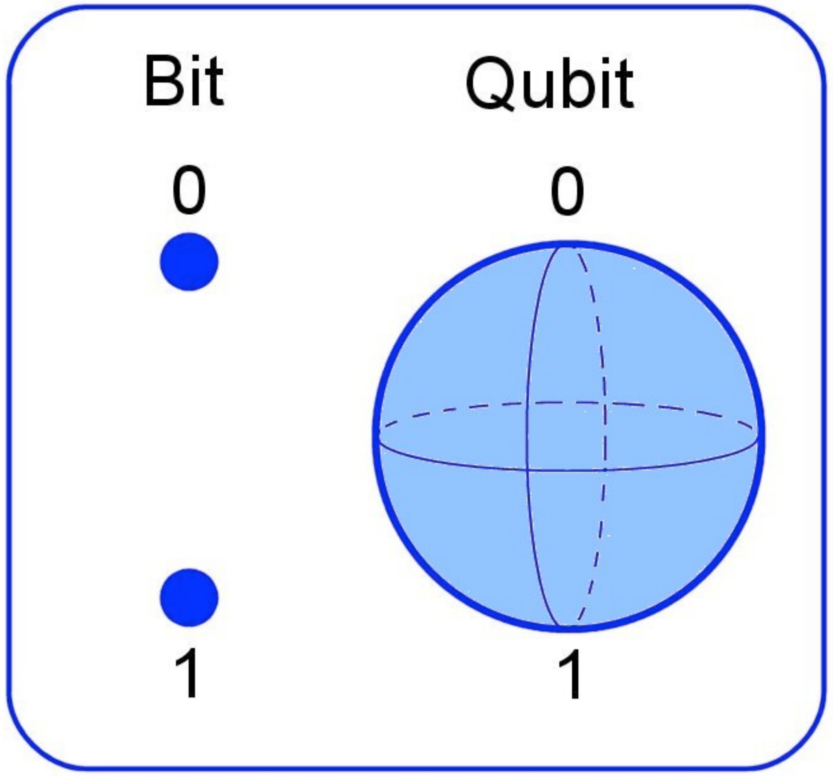

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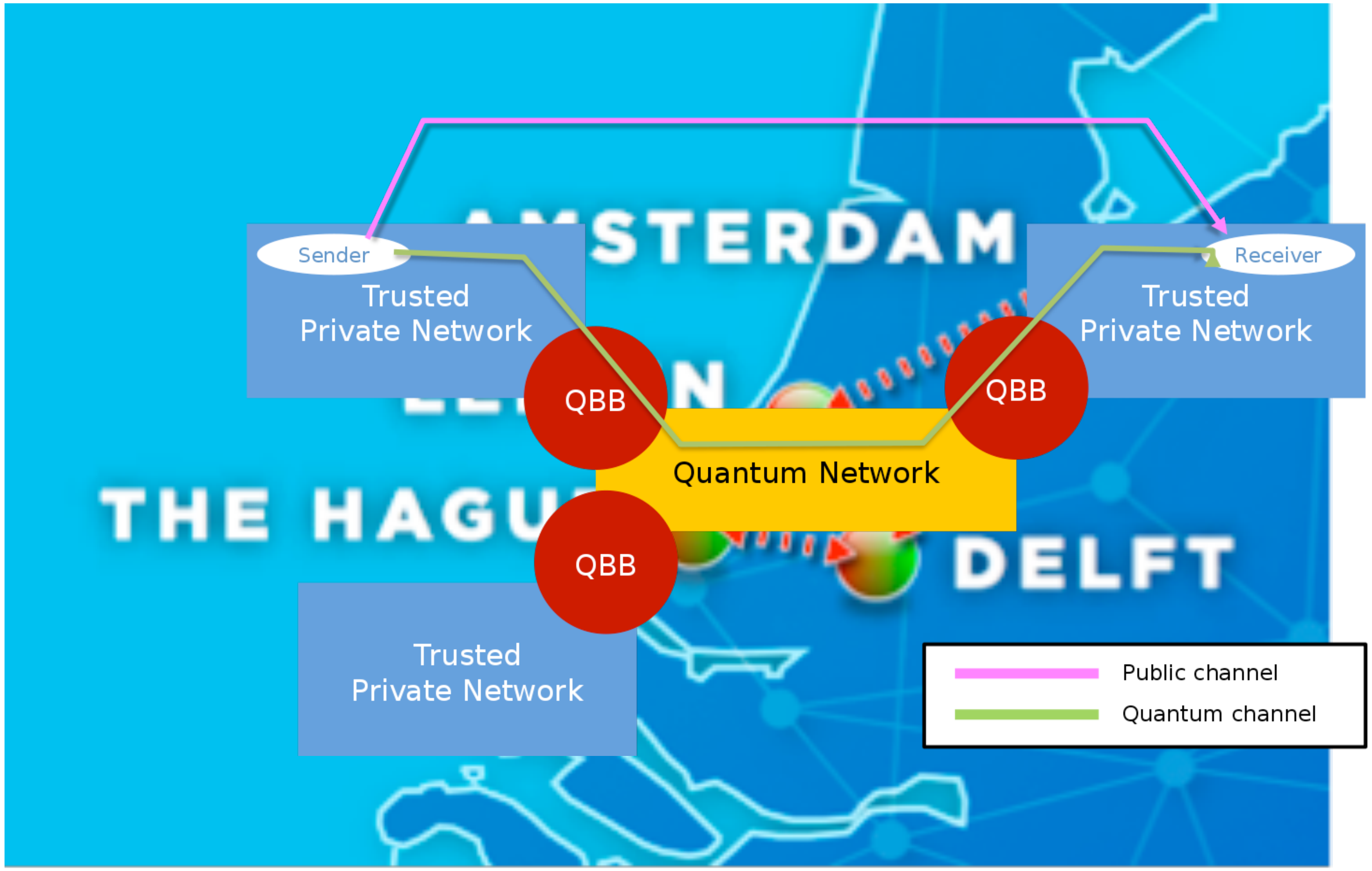

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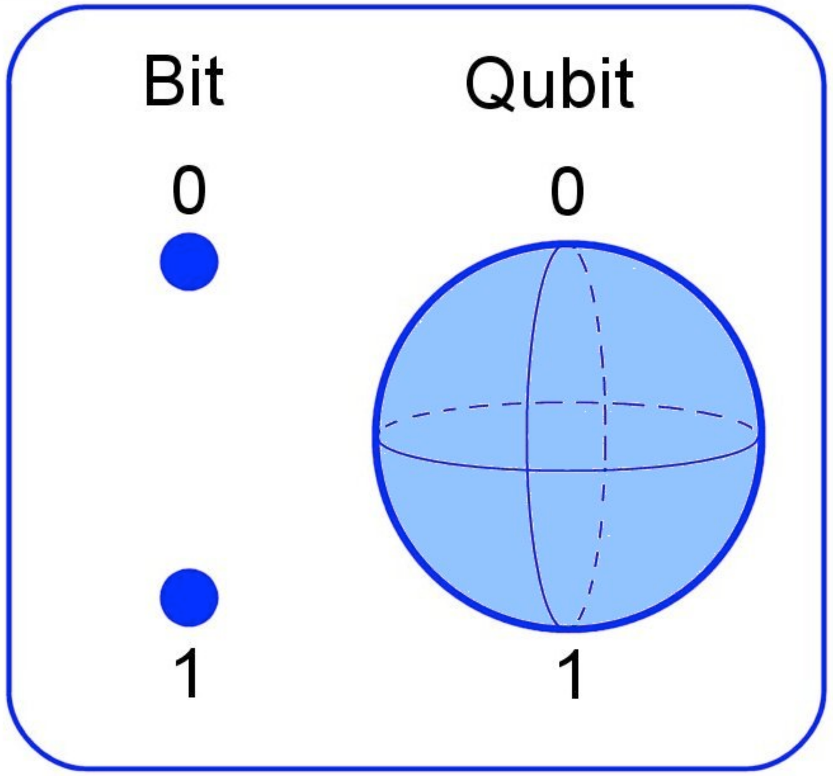

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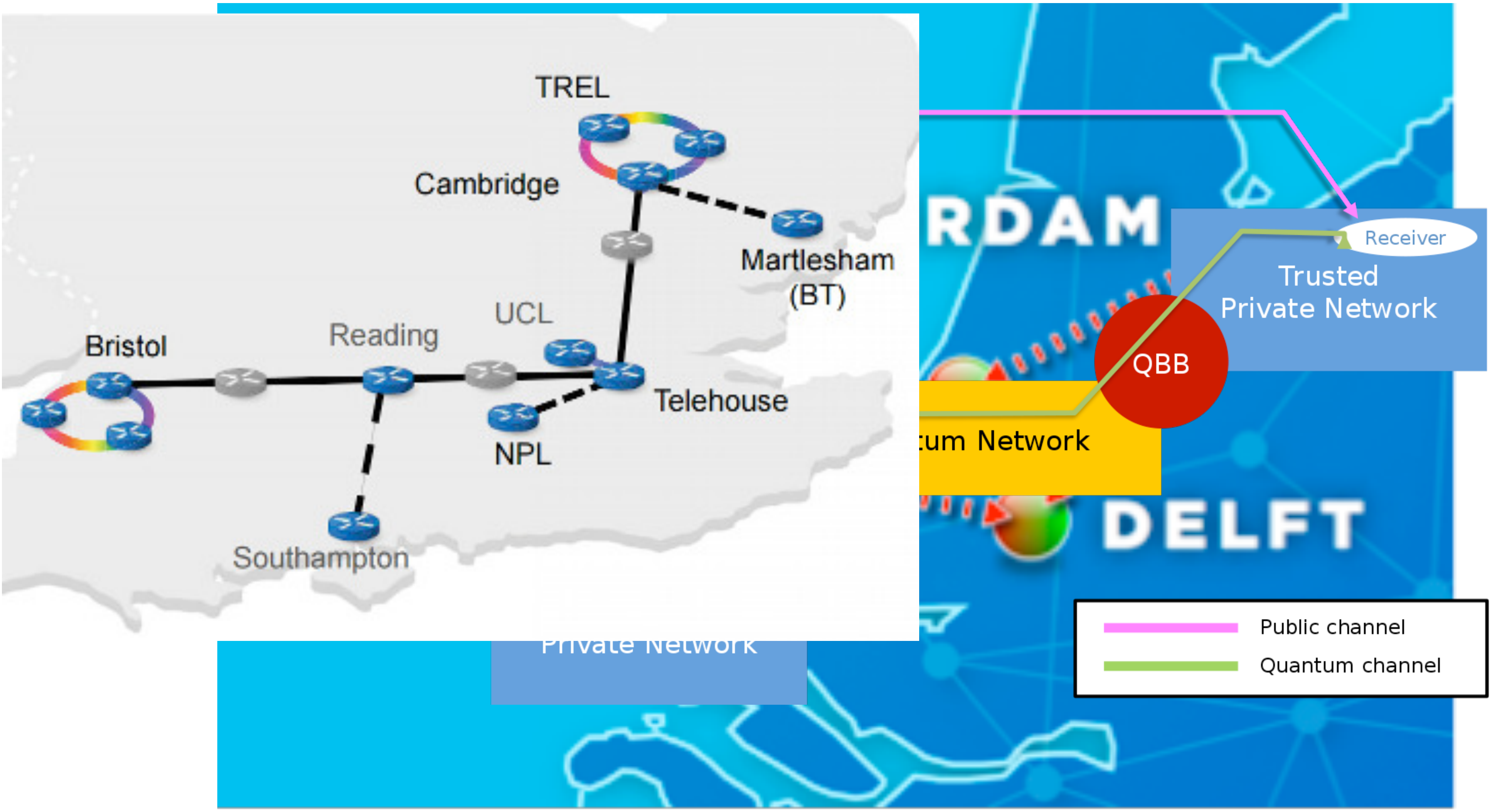

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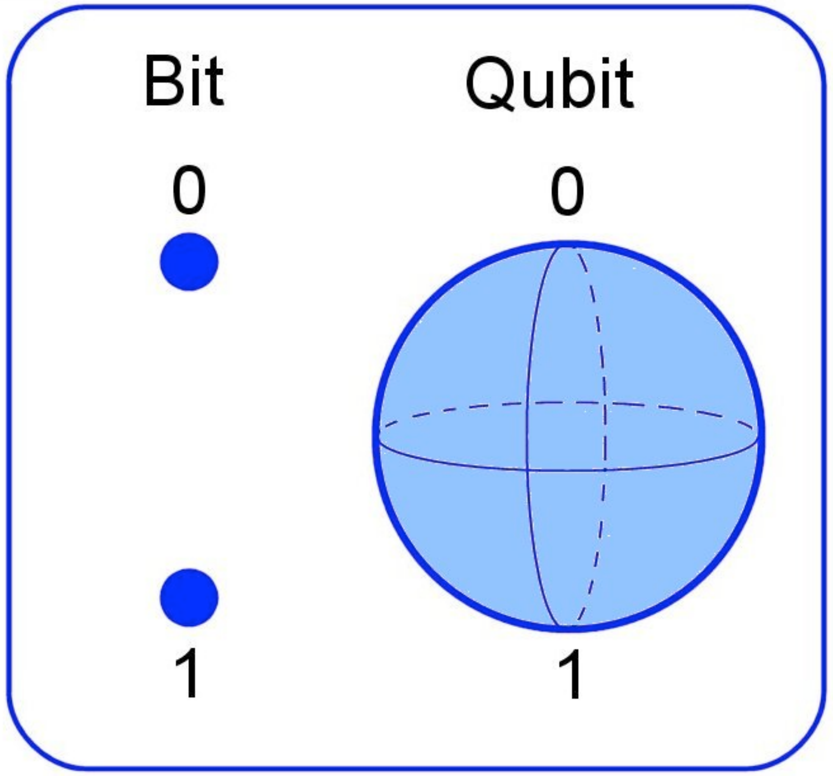

single qubits communications

Quantum Era

National Investments

Europe 1bn€
UK 270M £
Netherlands 80M $
US, Singapore,Canada

Quantum Machines

Private Investments

Google, IBM, Intel
Big VC founds
Startups Companies: D-Wave,IonQ, Rigetti

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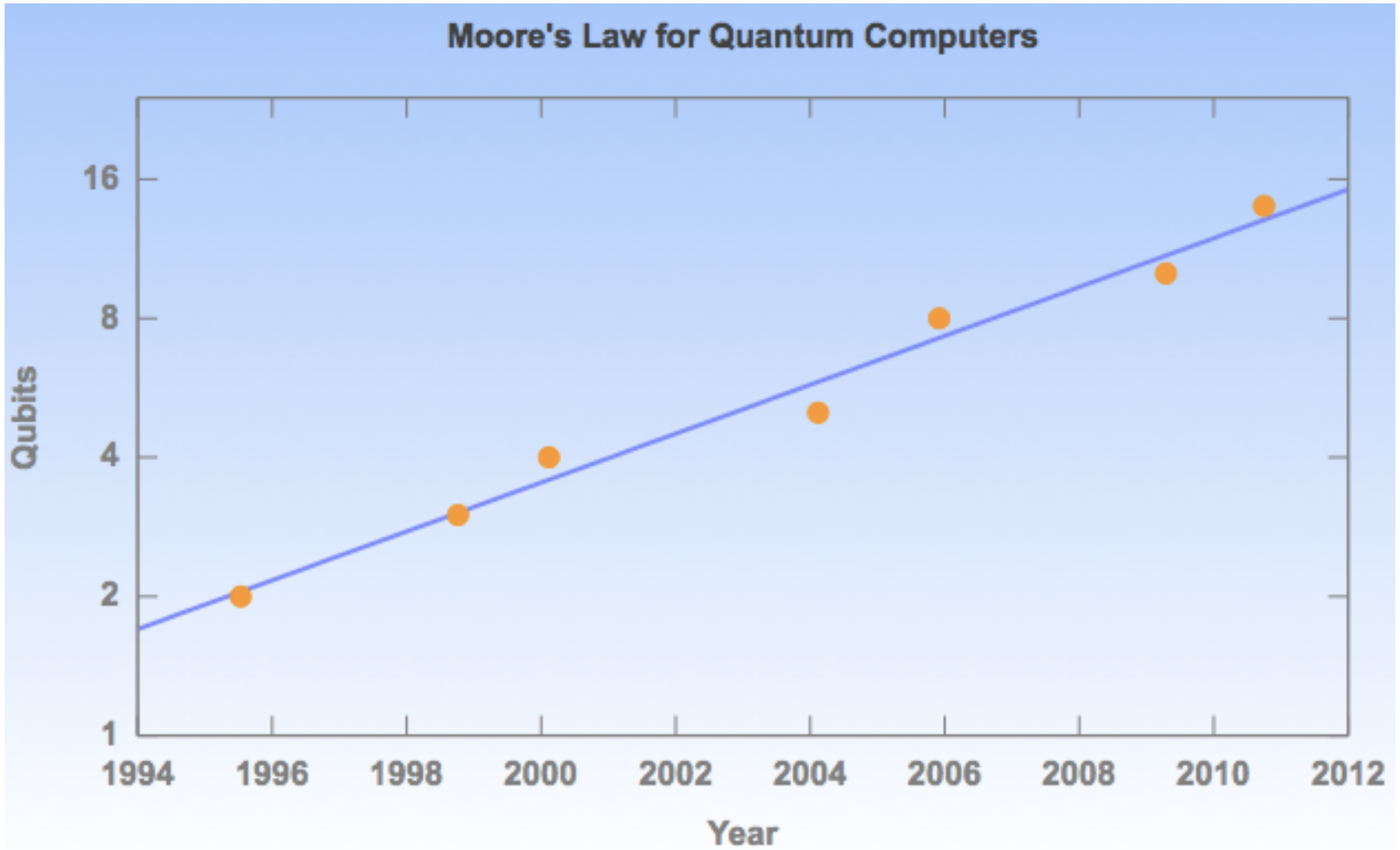

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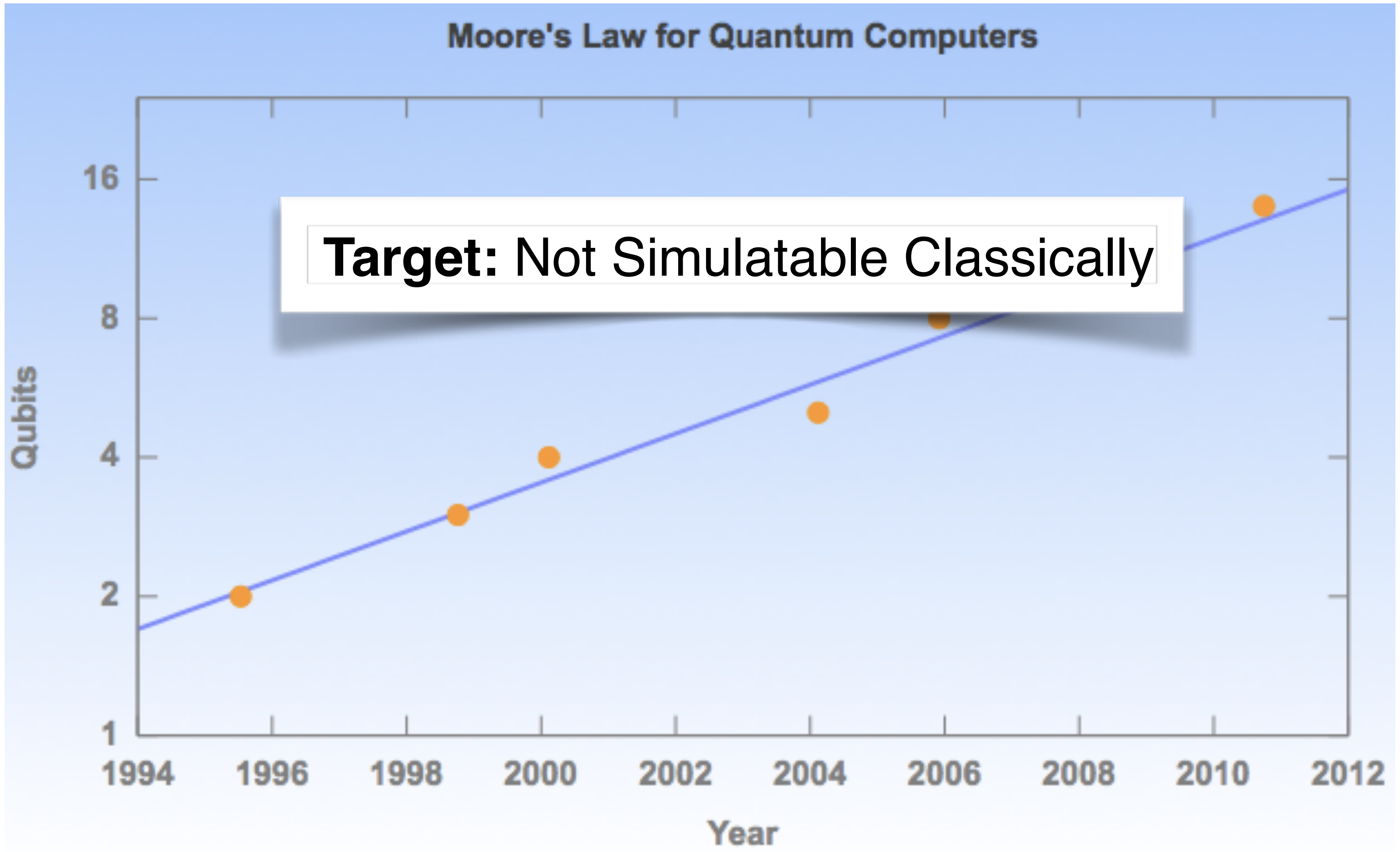
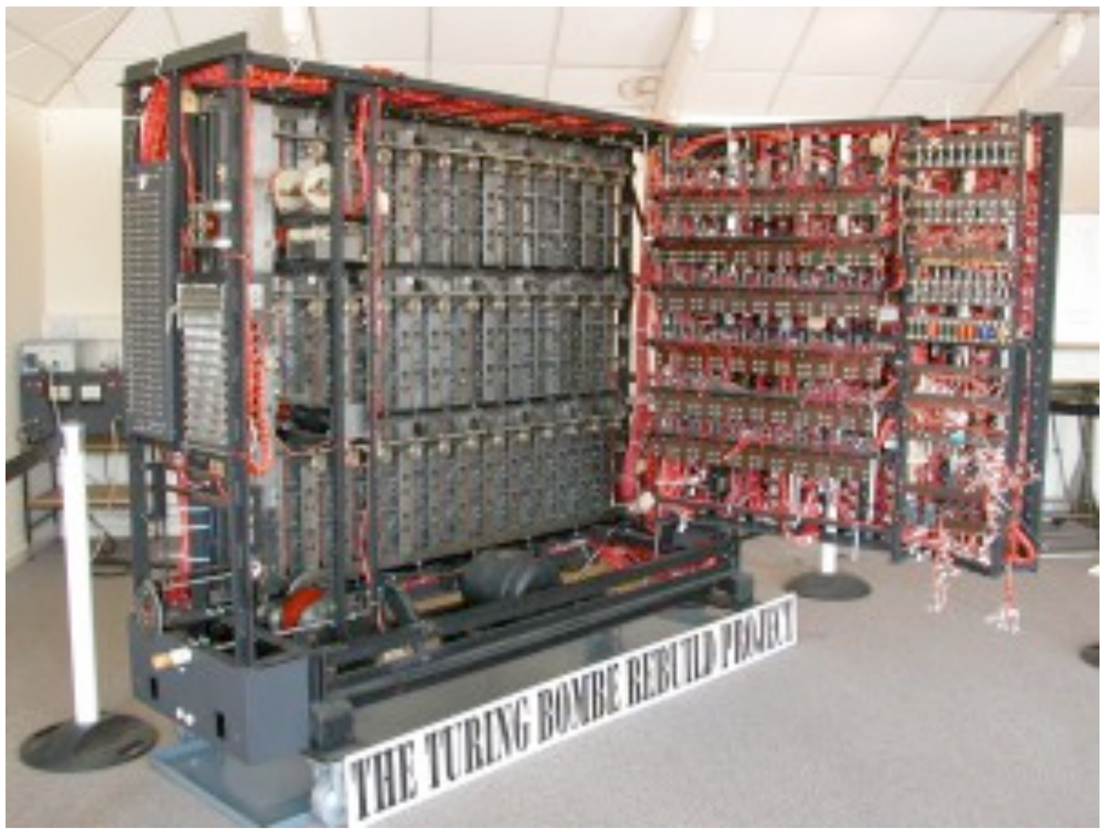
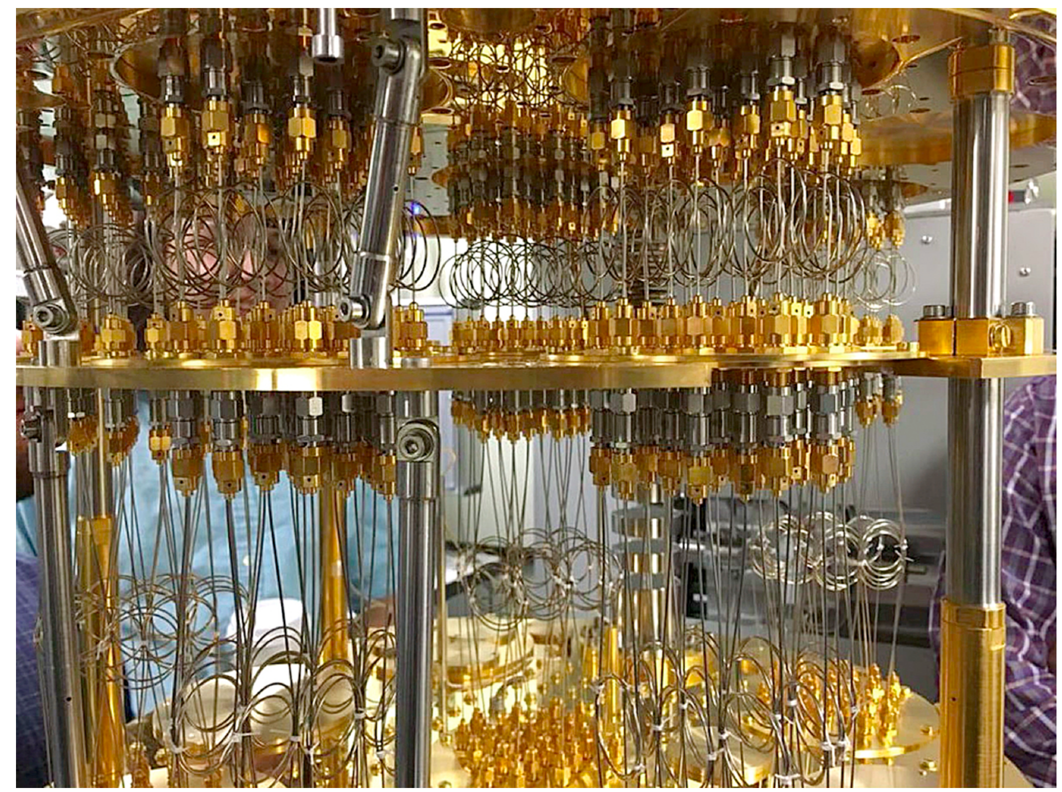


Target: Not Simulatable Classically

Turing Machine vs Quantum Machine





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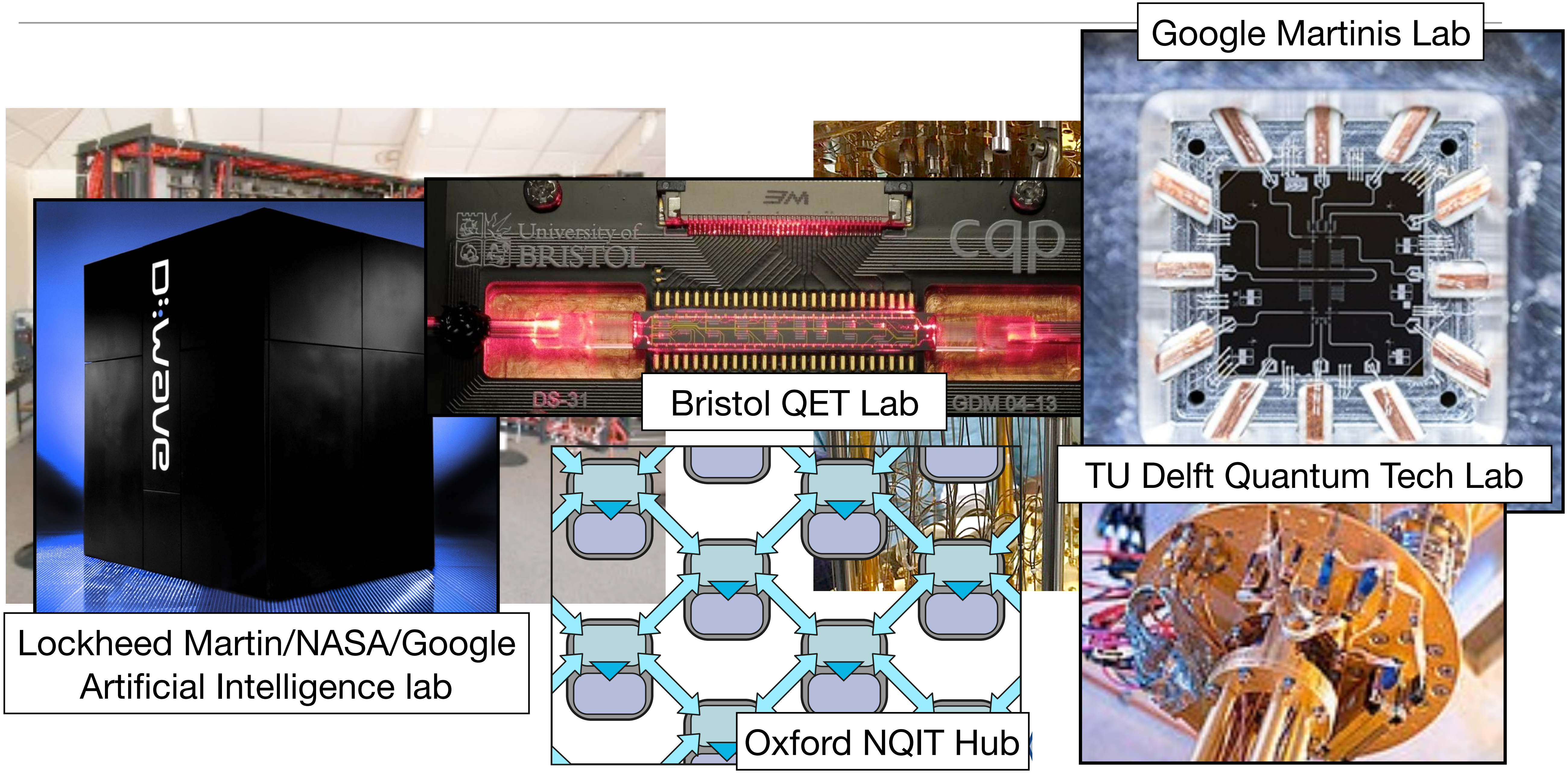
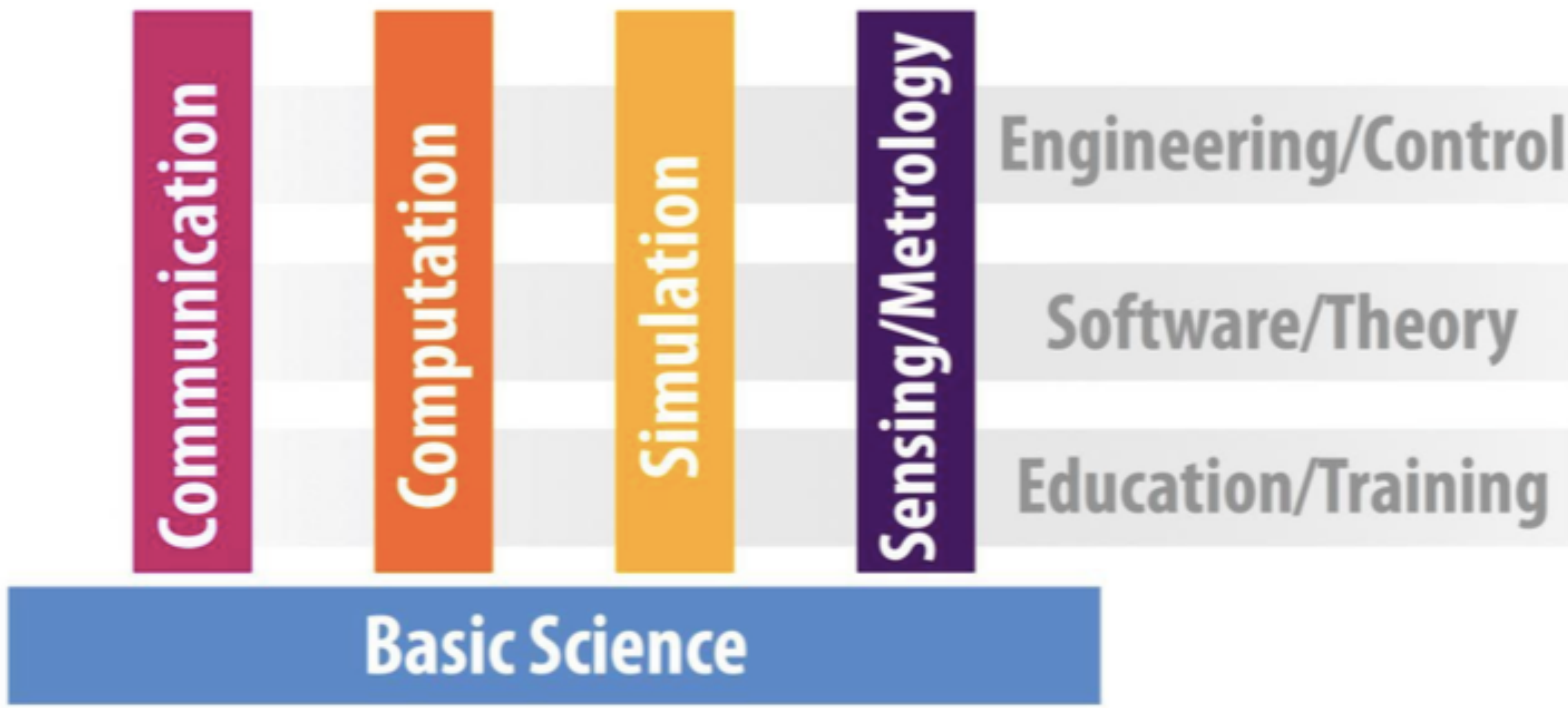
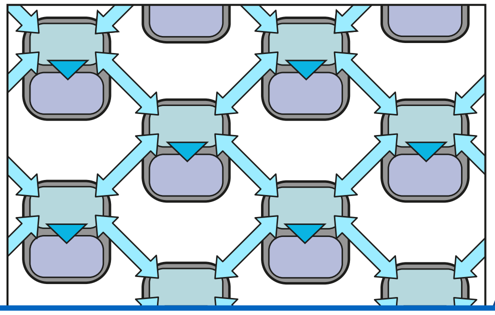


Quantum Flagship

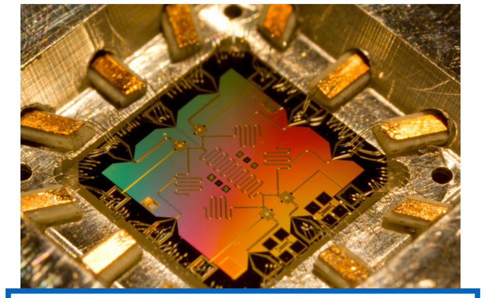


Vision of Quantum Technology



Communication Network

Hardware



Computing Device

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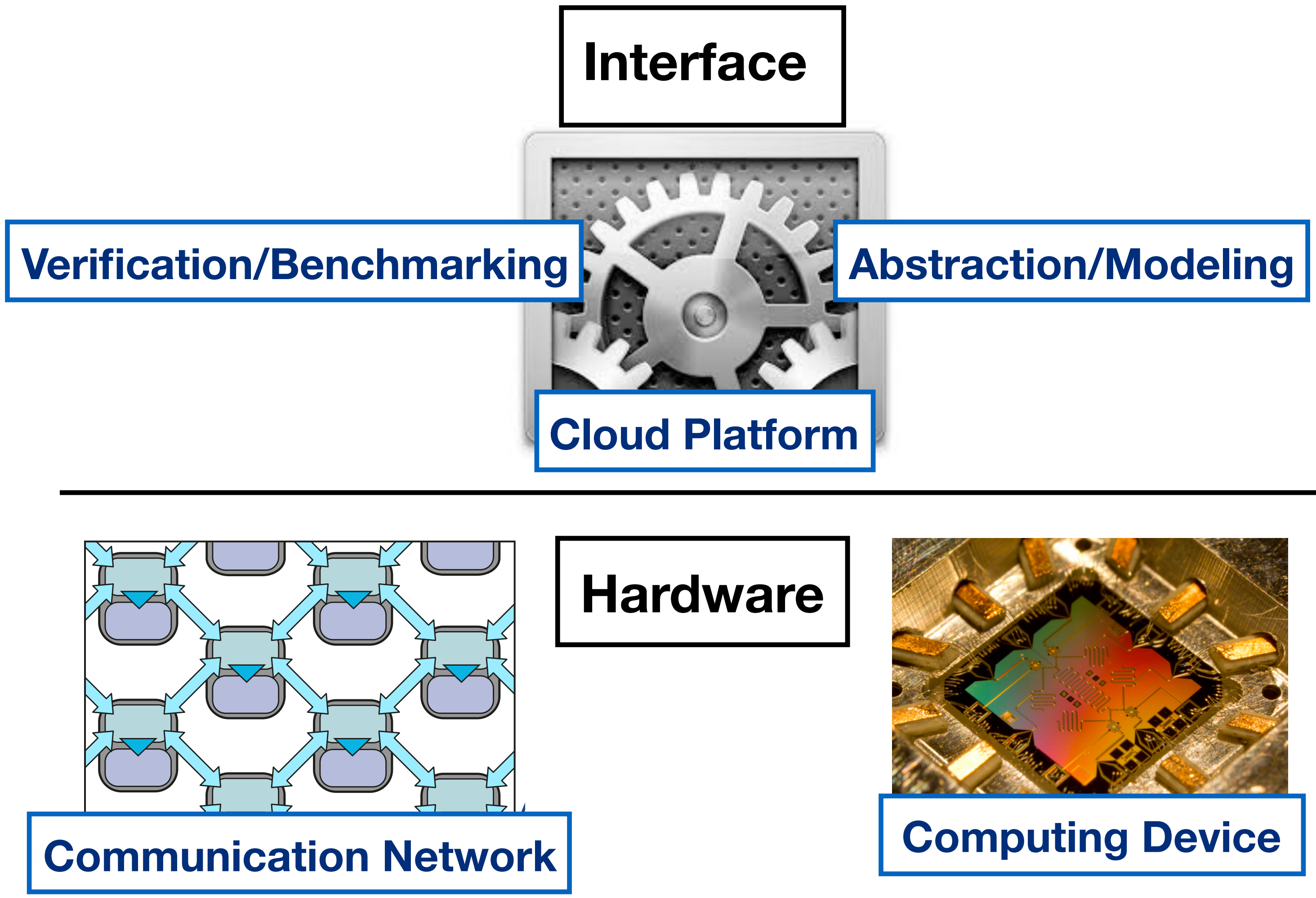

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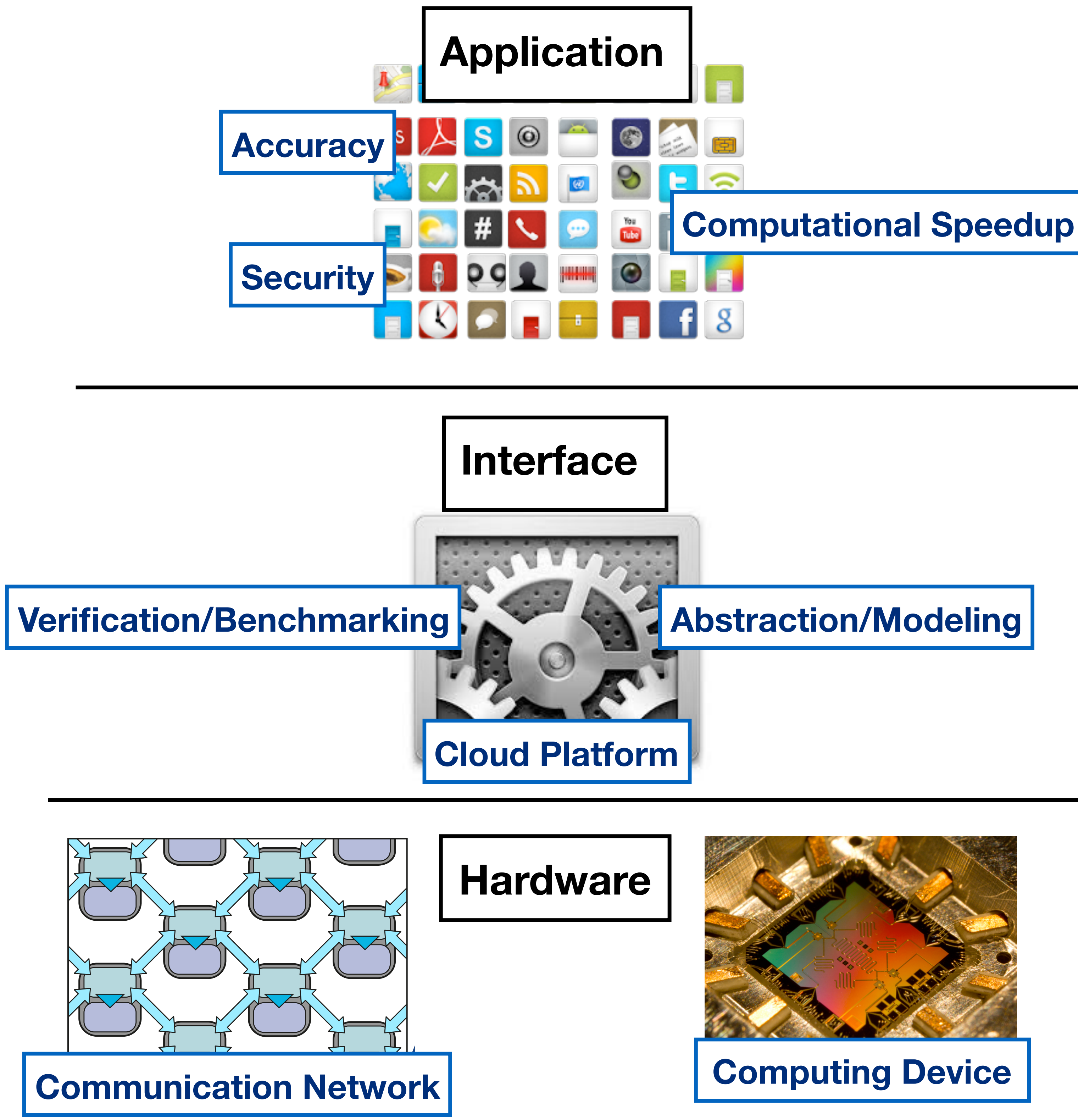

Quantum Internet

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- Secure communication
- Clock synchronisation
- Combining distant telescopes
- Communication Complexity Advantage
- Secure access to Quantum Cloud
- Bootstrapping small Quantum Computer

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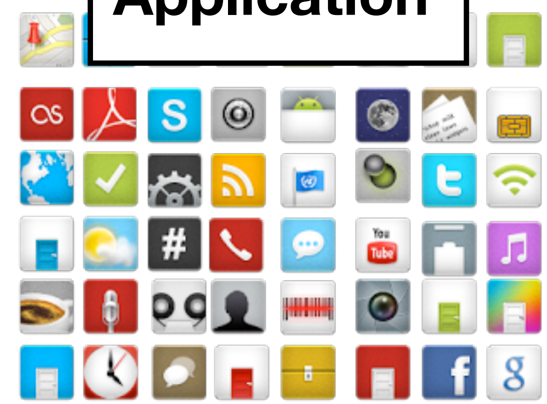

- Quantum Network Modules
- Network simulation and benchmarking
- Control Stack
- HAL Operating System
- Code optimization and compiling
- Bootstrapping small Quantum Computer

Interface

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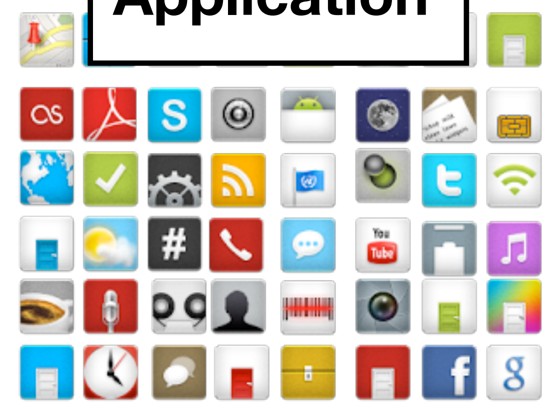

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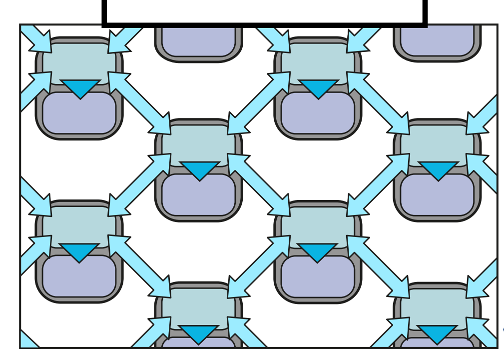
Interface

- Server and Client Nodes
- Hybrid Architecture
- Quantum Memory and Repeater
- Integration to long distance network

Hardware



Quantum Computing

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- Machine Learning
- Optimisation
- Quantum Chemistry

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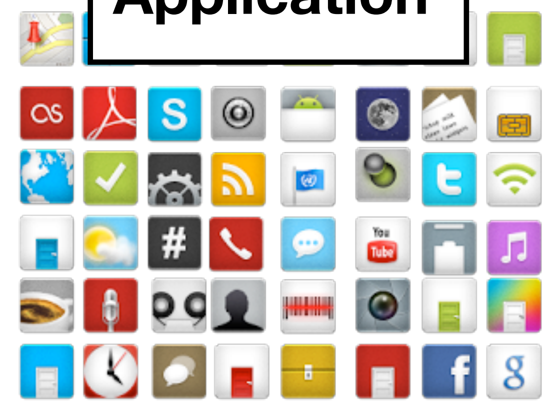

Interface

- Programming Language
- Verification
- HAL Operating System
- Code optimization and compiling
- Architecture Design

Quantum Computing

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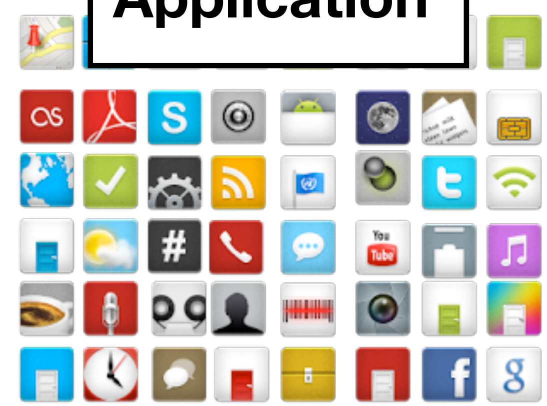

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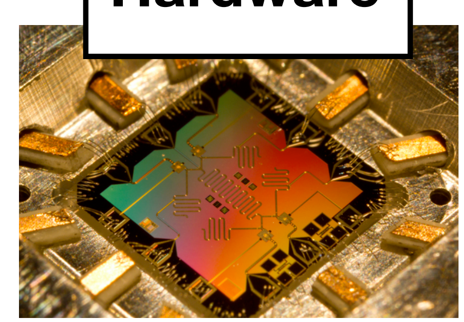
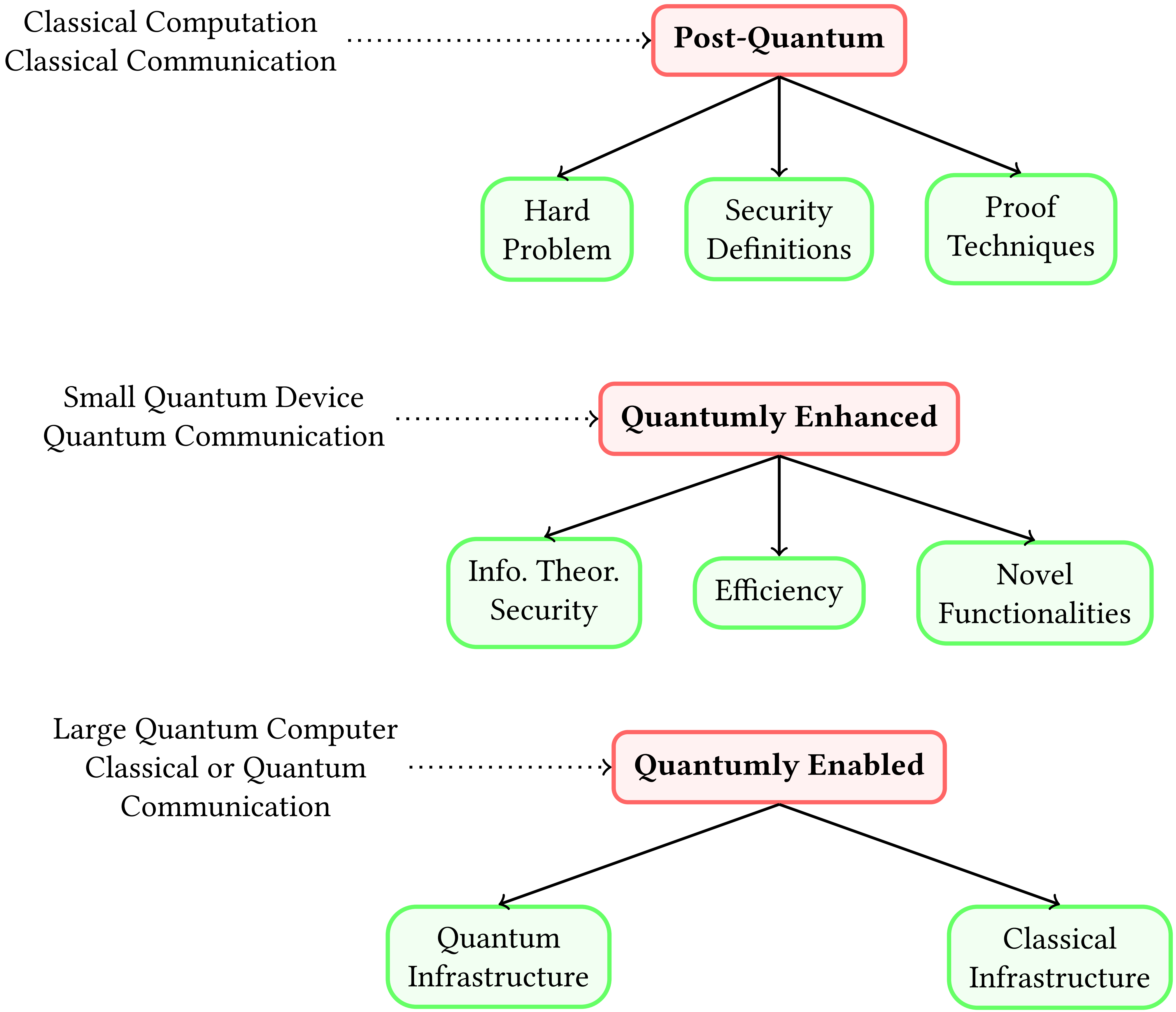


- Server and Client Nodes
- Hybrid Architecture
- Fault Tolerance
- Scaling


Classical Computation
Classical Communication
Post-Quantum
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Security Definitions
Proof Techniques
Small Quantum Device
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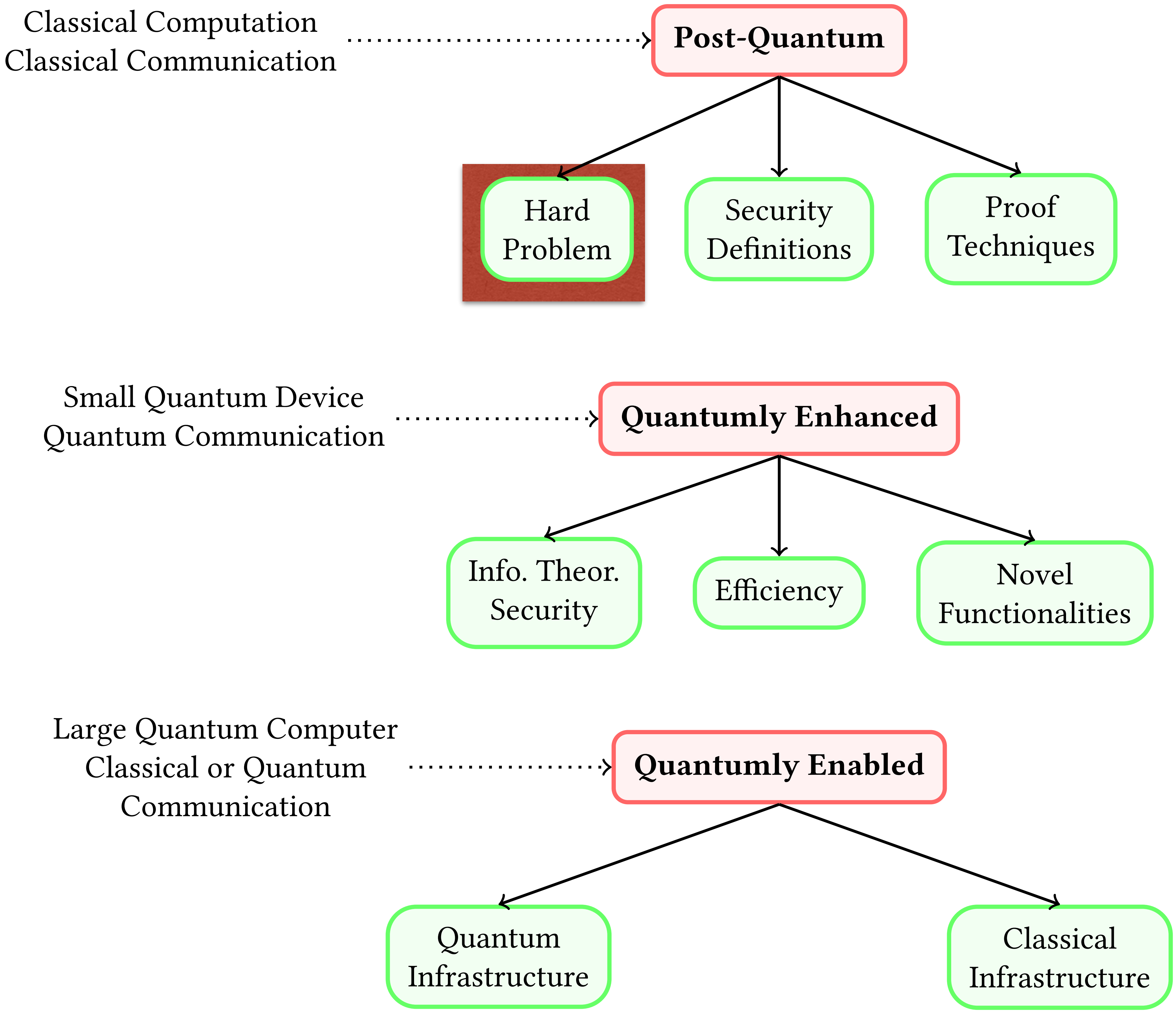
Classical Infrastructure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Quantum algorithms breaking computational assumptions

Factoring and Discrete Logarithm [Shor 94] Principal ideal problem [Hallgren 02]

Quantum effects breaking Information-theoretical assumptions

commitment scheme becomes non-binding [Crepeau,Salvail,Simard,Tapp 06]

Classical proof techniques no longer apply

rewinding

Post-Quantum

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Learning with Error (LWE)

as hard as worst-case lattice problems, believed to be exponentially hard against QC

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LWE-based Crypto Systems (FHE and etc)

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(classical) **mixed commitment schemes** (secure against quantum)

lifting classical security proof to the quantum setting, **coin flipping protocols**

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(classical) **mixed commitment schemes** (secure against quantum)

lifting classical security proof to the quantum setting, **coin flipping protocols**

(classical) **Zero-Knowledge Proof-of-Knowledge** (secure against quantum)

lifting classical security proof to the quantum setting, **secure function evaluation**

Algebraic Problems

Andrew M. Childs and Wim van Dam, 2008

The hidden subgroup problem

Let G be a finite Abelian group with group operations written additively

consider a function $f : G \to S$, where S is some finite set. We say that f *hides* the subgroup H

$f(x) = f(y)$ if and only if $x - y \in H$

find a generating set for H given the ability to query the function f

The hidden subgroup problem

Problem
Factorisation
Discrete log
Elliptic curve discrete log
Principal ideal
Shortest lattice vector
Graph isomorphism

The hidden subgroup problem

Problem	*Group*
Factorisation	$\mathbb{Z}$
Discrete log	$\mathbb{Z}_{p-1} \times \mathbb{Z}_{p-1}$
Elliptic curve discrete log	Elliptic curve
Principal ideal	$\mathbb{R}$
Shortest lattice vector	Dihedral group
Graph isomorphism	Symmetric group

The hidden subgroup problem

Problem	*Group*	*Cryptosystem*
Factorisation	$\mathbb{Z}$	RSA
Discrete log	$\mathbb{Z}_{p-1} \times \mathbb{Z}_{p-1}$	Diffie-Hellman, DSA,...
Elliptic curve discrete log	Elliptic curve	ECDH, ECDSA,...
Principal ideal	$\mathbb{R}$	Buchmann-Williams
Shortest lattice vector	Dihedral group	NTRU, Ajtai-Dwork,...
Graph isomorphism	Symmetric group	—

The hidden subgroup problem

Problem	*Complexity*	*Cryptosystem*
Factorisation	Polynomial[11]	RSA
Discrete log	Polynomial[11]	Diffie-Hellman, DSA,…
Elliptic curve discrete log	Polynomial[92]	ECDH, ECDSA,…
Principal ideal	Polynomial[93]	Buchmann-Williams
Shortest lattice vector	Subexponential[94,95]	NTRU, Ajtai-Dwork,…
Graph isomorphism	Exponential	—

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Quantum algorithms: an overview

Ashley Montanaro

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Fourier transforms over finite Abelian groups

The hidden subgroup problem

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Andrew M. Childs and Wim van Dam, 2008

Fourier transforms over finite Abelian groups

$$|x\rangle \mapsto \frac{1}{\sqrt{N}} \sum_{y \in \mathbb{Z}/N\mathbb{Z}} \omega_N^{xy} |y\rangle,$$

$$\omega_N := e^{2\pi i/N}$$

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one-dimensional irreducible representations

$$\psi : G \to \mathbb{C} \quad \psi(a+b) = \psi(a)\psi(b)$$

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Efficient quantum circuit for the QFT

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$$F_{\mathbb{Z}/N\mathbb{Z}} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega_N & \omega_N^2 & \cdots & \omega_N^{N-1} \\ 1 & \omega_N^2 & \omega_N^4 & \cdots & \omega_N^{2N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_N^{N-1} & \omega_N^{2N-2} & \cdots & \omega_N^{(N-1)(N-1)} \end{pmatrix}$$

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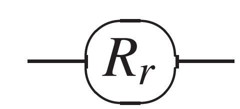
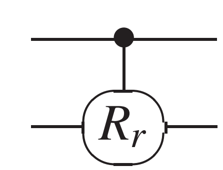
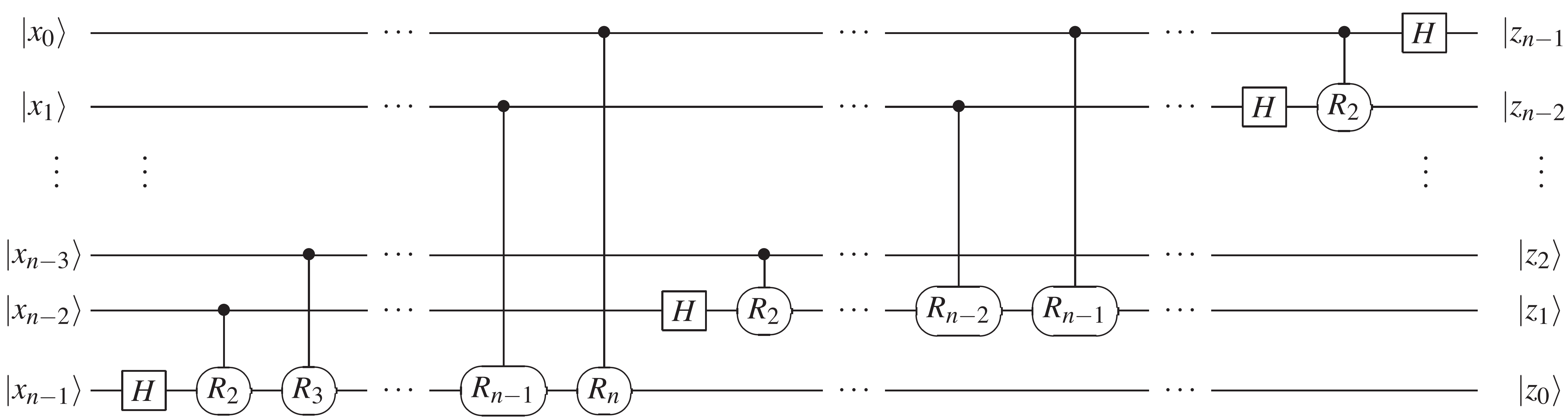
$$R_r := \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{2\pi \mathrm{i}/2^r} \end{pmatrix} \simeq$$



$$\Lambda(R_r) := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mathrm{e}^{2\pi \mathrm{i}/2^r} \end{pmatrix} \simeq$$



QFT



An efficient (size $O(n^2)$) quantum circuit for the quantum Fourier transform over $Z/2^nZ$

Period Finding Over Z/NZ

$$\boxed{f : \mathrm{Z}/N\mathrm{Z} \rightarrow S \ \text{ with period } r}$$

$$f(x) = f(y) \text{ if and only if } \frac{x-y}{r} \in \mathbb{Z}$$

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We can find the period r efficiently using the HSP over the additive group $\mathrm{Z}/N\mathrm{Z}$.

Represent $x \in \mathrm{Z}/N\mathrm{Z}$ uniquely as an integer $x \in \{0, \ldots, N-1\}$

The irreducible representations $\psi : \mathrm{Z}/N\mathrm{Z} \rightarrow \mathrm{C}$ can be labeled by integers $y \in \{0,\ldots,N-1\}$

Period Finding Over Z/NZ - Algorithm

1. Apply the Fourier transform over Z/NZ to the state $|0\rangle$

$$|\mathbb{Z}/N\mathbb{Z}\rangle = \frac{1}{\sqrt{N}} \sum_{x \in \mathbb{Z}/N\mathbb{Z}} |x\rangle$$

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2. Query the function f in an ancilla register

$$\frac{1}{\sqrt{N}} \sum_{x \in \mathbb{Z}/N\mathbb{Z}} |x, f(x)\rangle$$

Period Finding Over Z/NZ - Algorithm

3. Measure the ancilla register.

The first register will be in a superposition of those x consistent with the observed function value.

Period Finding Over Z/*N*Z - Algorithm

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$$\sqrt{\frac{r}{N}}\sum_{j=0}^{\frac{N}{r}-1}|s+jr\rangle$$

Period Finding Over Z/NZ - Algorithm

3. Measure the ancilla register.

The first register will be in a superposition of those x consistent with the observed function value.

$$\sqrt{\frac{r}{N}}\sum_{j=0}^{\frac{N}{r}-1}|s+jr\rangle$$

for unknown offset $s \in \{0,\ldots,r-1\}$ *corresponding to the uniformly random observed function value* $f(s)$

Period Finding Over Z/NZ - Algorithm

4. Apply the Fourier transform over $\mathbb{Z}/N\mathbb{Z}$

Period Finding Over Z/NZ - Algorithm

4. Apply the Fourier transform over Z/*N*Z

$$\sqrt{\frac{r}{N}} \sum_{y \in \mathbb{Z}/N\mathbb{Z}} \sum_{j=0}^{\frac{N}{r}-1} \omega_N^{(s+jr)y} |y\rangle.$$

Period Finding Over Z/NZ - Algorithm

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Period Finding Over Z/NZ - Algorithm

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giving kN/r and hence the fraction k/r

which, when reduced to lowest terms, has r/gcd(r,k) as its denominator.

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$$\prod_{p\ prime}\left(1-\frac{1}{p^2}\right)=6/\pi^2\approx 0.61$$

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Given oracle $f:\{0, 1\}^n \rightarrow \{0, 1\}$, find x such that $f(x) = 1$

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Quantum algorithms: an overview

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Finding the minimum of an unsorted list of N integers

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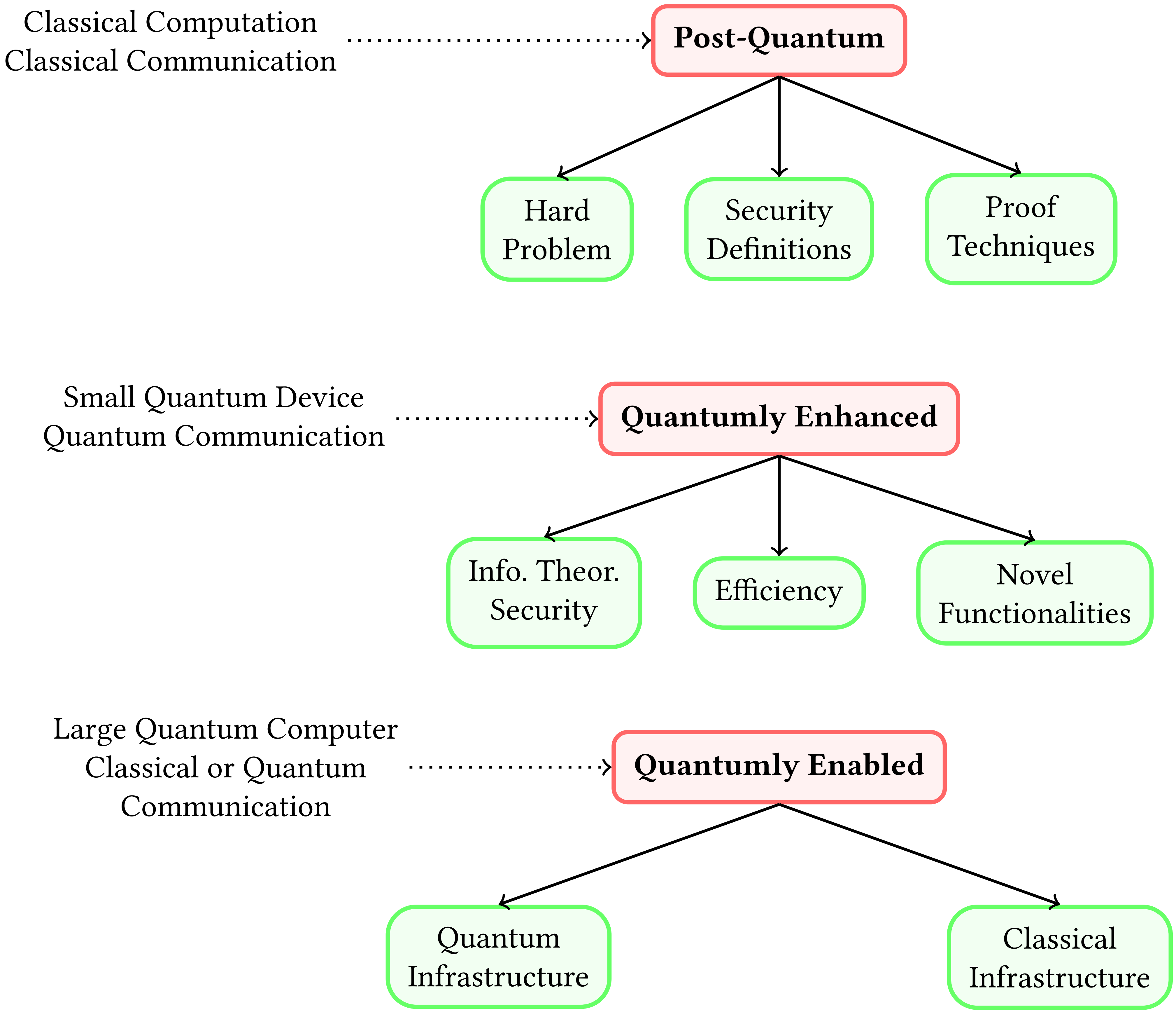
Solving systems of boolean multivariate quadratic equations

Input. $f_1(x_1, \ldots, x_n), \ldots, f_m(x_1, \ldots, x_n) \in \mathbb{F}_2[x_1, \ldots, x_n]$.
Goal. Find – if any – a vector $(z_1, \ldots, z_n) \in \mathbb{F}_2^n$ such that:

$$f_1(z_1, \ldots, z_n) = 0, \ldots, f_m(z_1, \ldots, z_n) = 0.$$

combine Grover's technique with a Grobner basis-based algorithm

$$O(2^{0.47n})$$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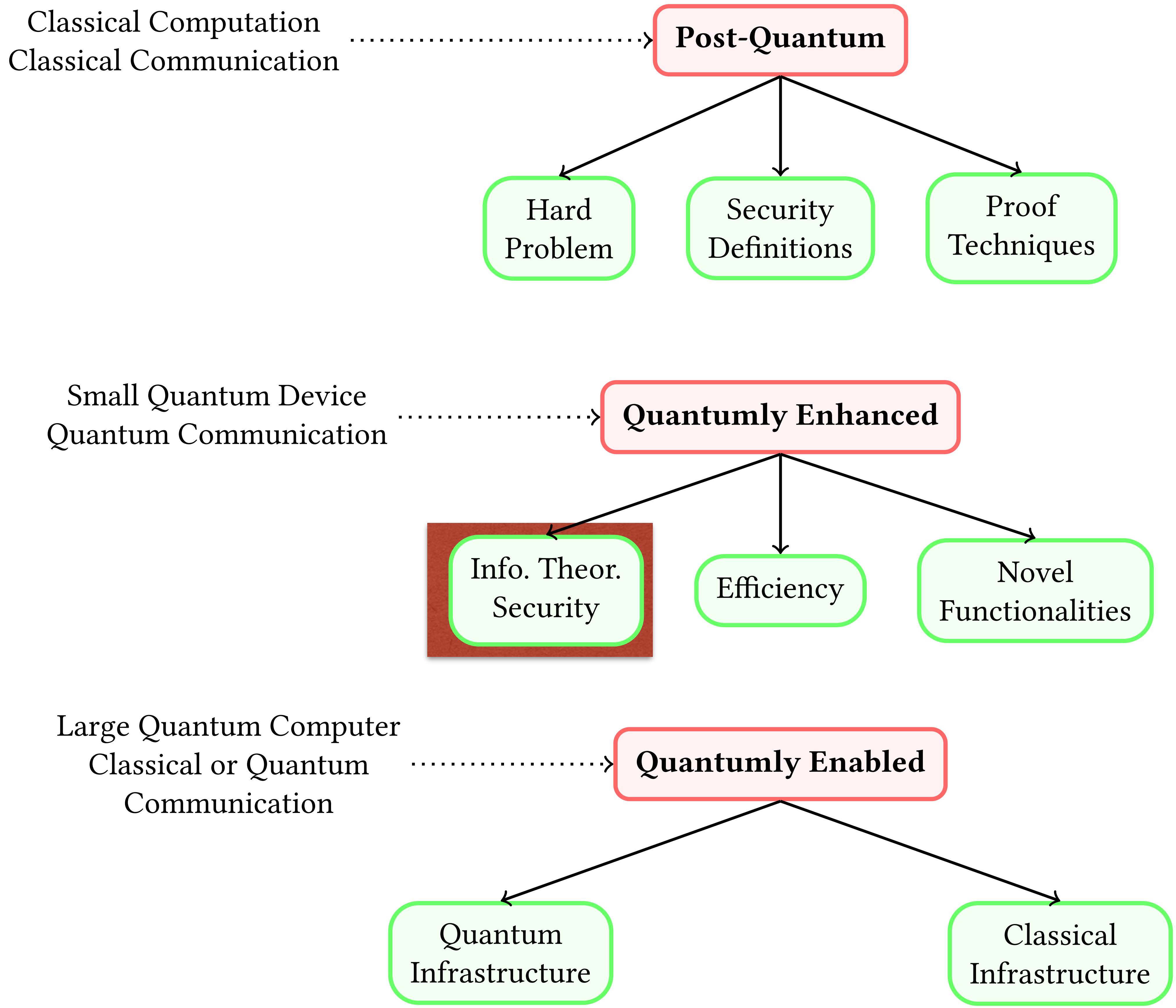
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qubits transmissions and classical post-processing

↓

unconditional security based on physical laws

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Information gain vs. disturbance No Cloning Spooky actions at a distance

History

1970 - **quantum money** (Wiesner)

The first link between secrecy and quantum physics

The bill contains photons that bank "polarised" in random directions

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1997 - **bit commitment and oblivious transfer** (Lo and Chau, Mayers)
contrary to the case of QKD and secret sharing
quantum physics cannot guarantee unconditional security

History

2007 - **bounded-storage models** (Damgaard et al; Wehner, Schaffner, Terhal)

unconditionally secure OT and BC is possible

where honest parties need no quantum memory, whereas an adversarial must store

at least n/2 qubits to break the protocol, where n is the number of qubits

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2009 - **blind quantum computing** (Broadbent, Fitzsimons and Kashefi)
Unconditionally secure quantum delegated cmputing
with implementation (Barz, et.al. 2012)

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Unconditionally secure authentication of the classical channel
requires Alice and Bob to pre-share an initial secret key
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QKD therefore does not create a secret key out of nothing:
it will expand a short secret key into a long one,
so strictly speaking it is a way of **key-growing**

Bennett Brassard - on paper

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Alice prepares a photon in one of the four states and sends it to Bob

Bob measures it in either the + or the × basis

This step is repeated N times. Both Alice and Bob have a list of N pairs **(bit,basis)**

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Alice and Bob reveal a random sample of their raw keys and estimate the error rate
They have to correct them and to erase the information that Eve obtains by
communication on the classical channel, **(classical post-processing)**
Alice and Bob share either a secret key or abort

Security

a non-secret key is never used

Either the authorised partners can create a secret key (a common list of secret bits known only to themselves), or they **abort** the protocol.

After classical communication Alice and Bob estimate
how much information about their lists of bits has leaked out to Eve
Such an estimate is impossible in classical communication.

In a quantum channel, leakage of information is quantitatively related to a perturbation of the communication.

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fundamental principles of quantum physics

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Quantum correlations obtained by separate measurements
on entangled pairs violate ***Bell's inequalities***
They cannot be created by pre-established agreement
The outcomes of the measurements did not exist before the measurements
but then, in particular, Eve could not know them.

Post processing

(In the absence of system errors) the spy will get detected
by the errors she induces in the communication

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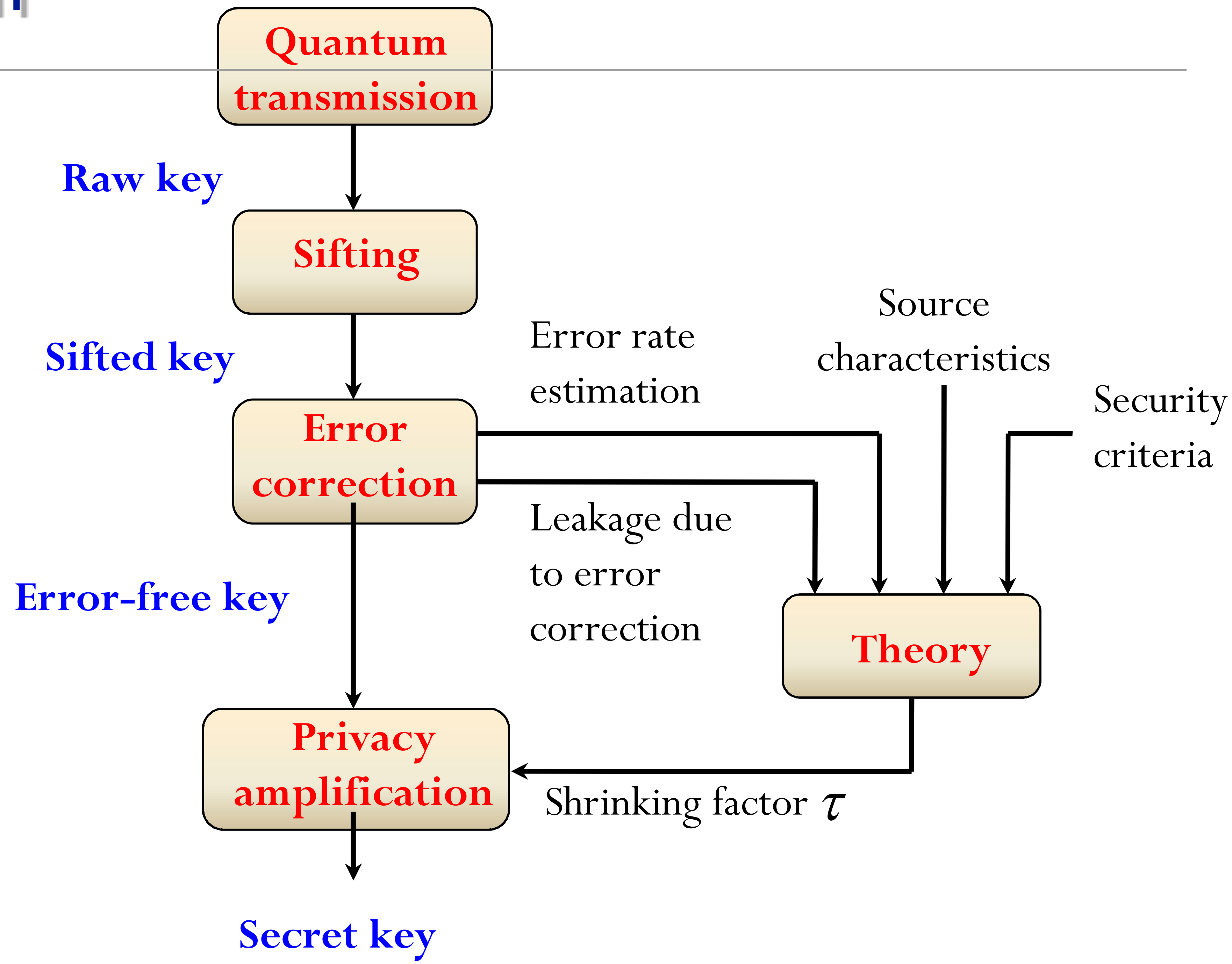
A complete QKD protocol should consider all errors as errors due to Eve, take into account possible information leakage, and bound this leakage as a function of the error rate

this is performed by two additional processes

Error correction + Privacy amplification

both are classical procedures

A QKD algorithm



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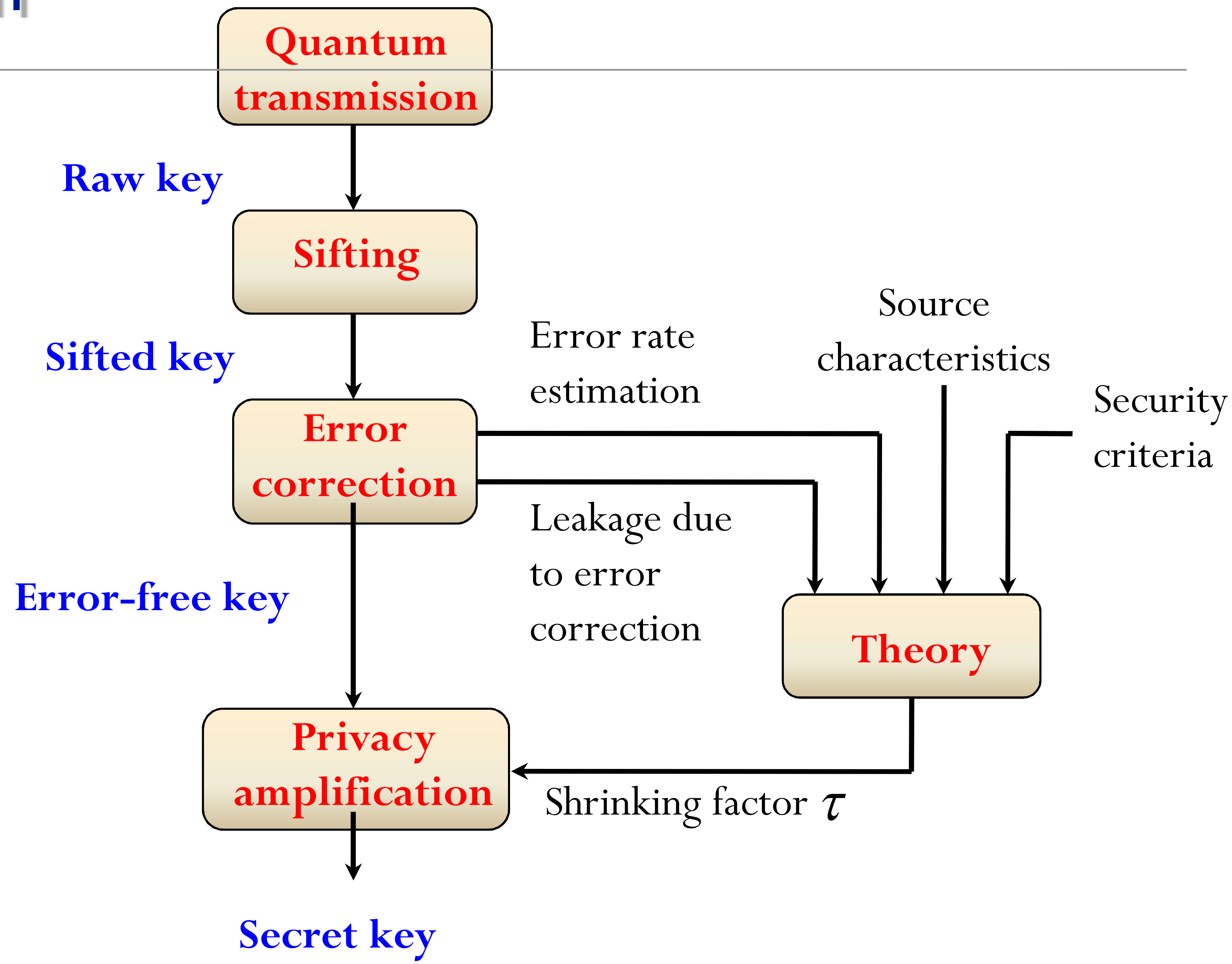

A security proof of a QKD protocol, which provides a given **shrinking factor** is a very difficult theoretical exercise with still many open questions

Composable security

A composable definition of security is the one based on the trace-norm (Ben-Or et al., 2005; Renner and Konig, 2005):

$$\frac{1}{2}\|\rho_{\mathcal{K}E} - \tau_{\mathcal{K}} \otimes \rho_E\|_1 \leq \varepsilon$$

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This is an extension of simulation-based definitions of universally composable security

Trace-norm contracts under QM transformations
plays the role of “statistical distance” or total variation

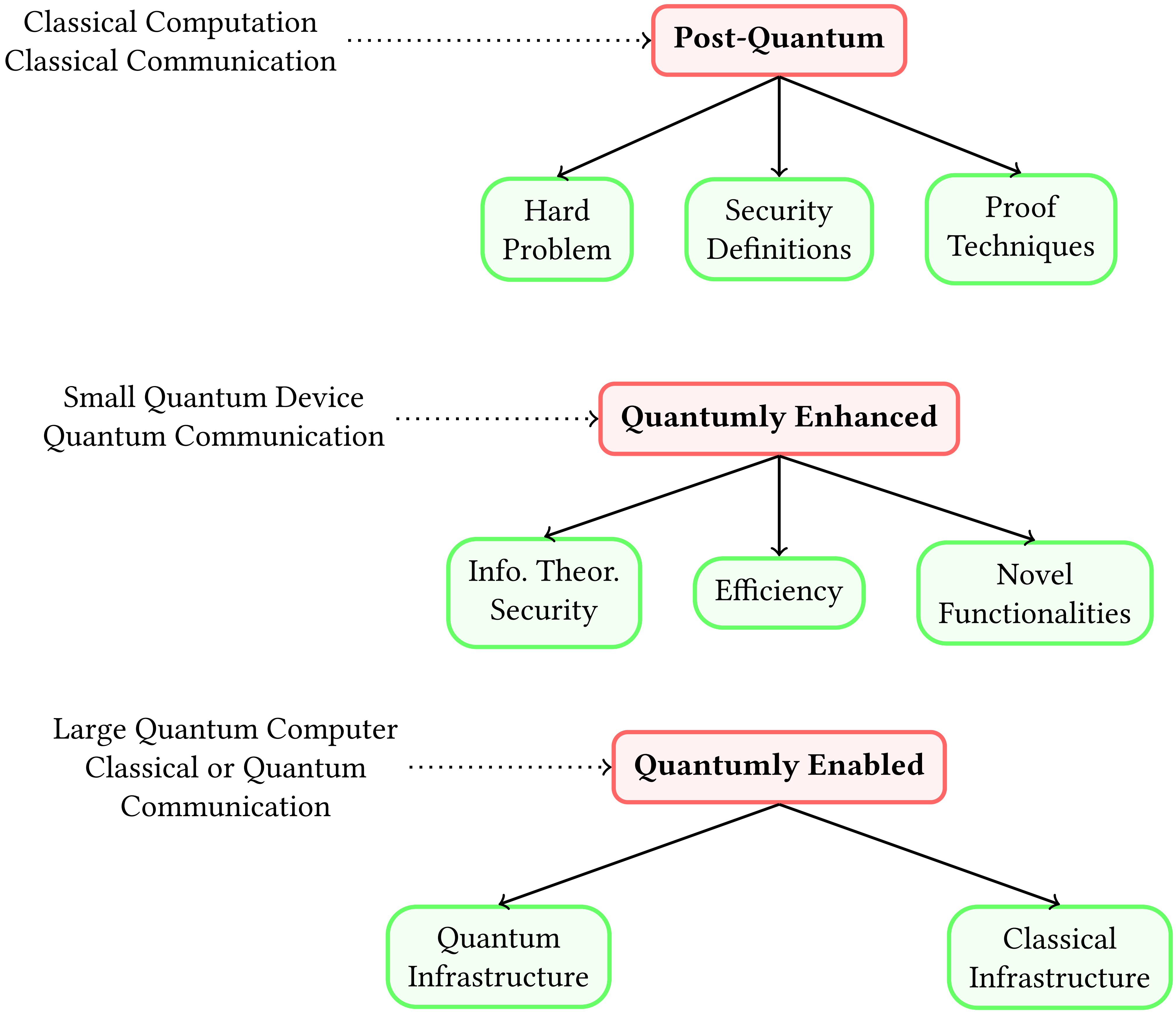
Security property - finally!

$$\frac{1}{2}\|\rho_{\mathcal{K}E} - \tau_{\mathcal{K}} \otimes \rho_E\|_1 \leq \varepsilon$$

the security requirement holds with high probability

$$\mathrm{Prob}\left[\|\rho_{\mathcal{K}E} - \tau_{\mathcal{K}} \otimes \rho_E\|_1 > 2\varepsilon\right] \lesssim e^{\ell - F(\rho_{\mathcal{K}E},\varepsilon)}$$

concretely, *F* will be depending on the protocol, and gives the length *l* of the secret key that can be extracted as a function of the indistinguishability/security parameter ε for a certain level of risk



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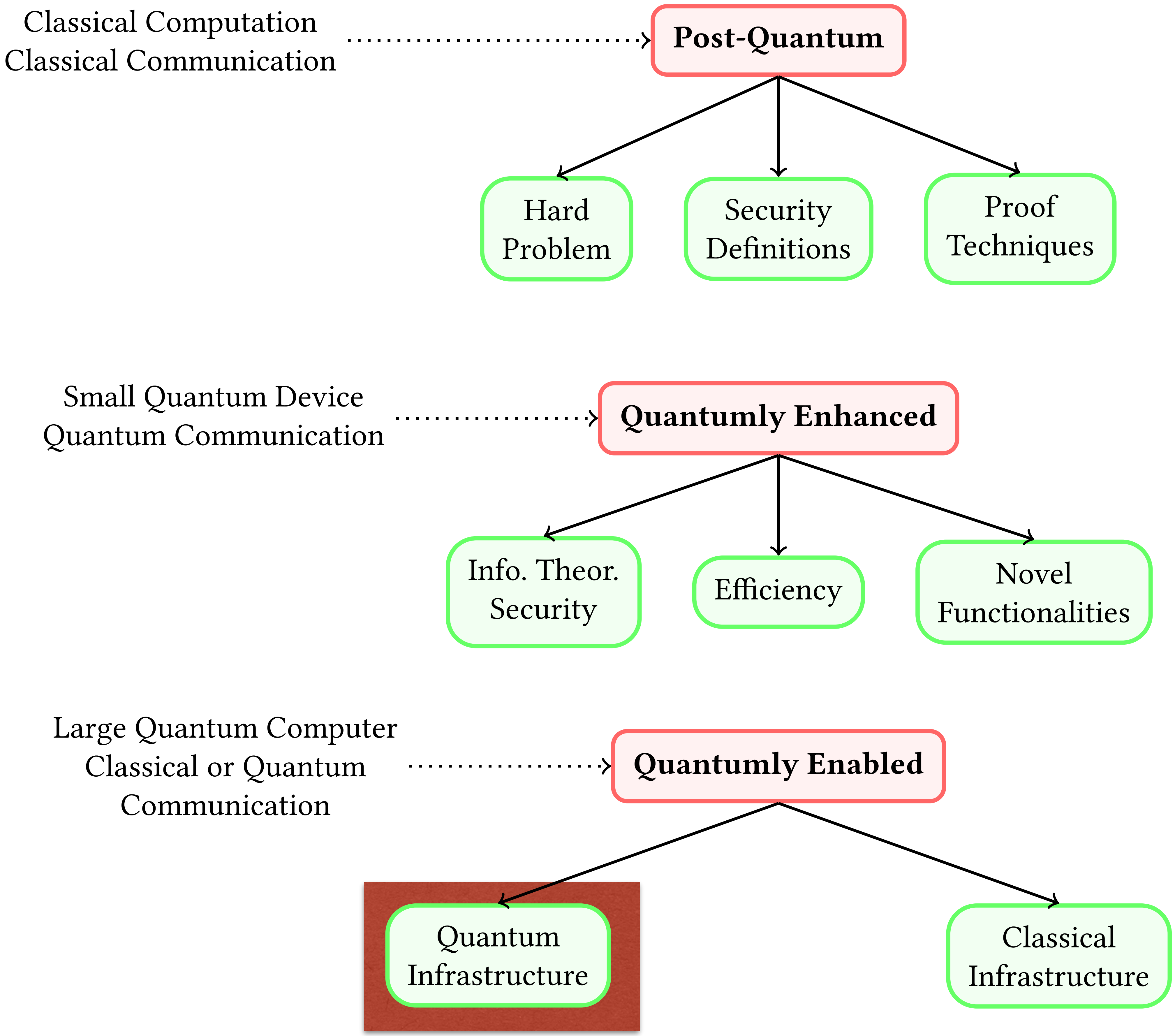
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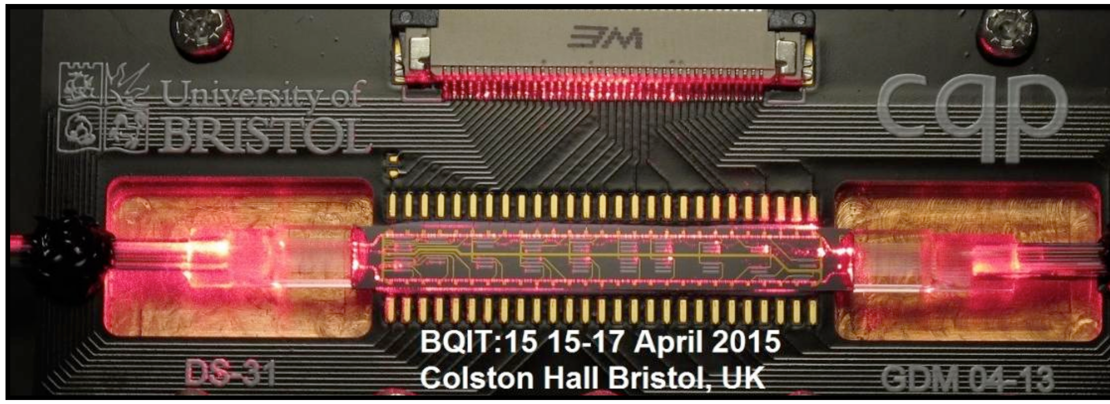
Quantum Cloud Service



Quantumly Enabled

Quantum Delegated Computing

Quantum Yao Garbled Circuit

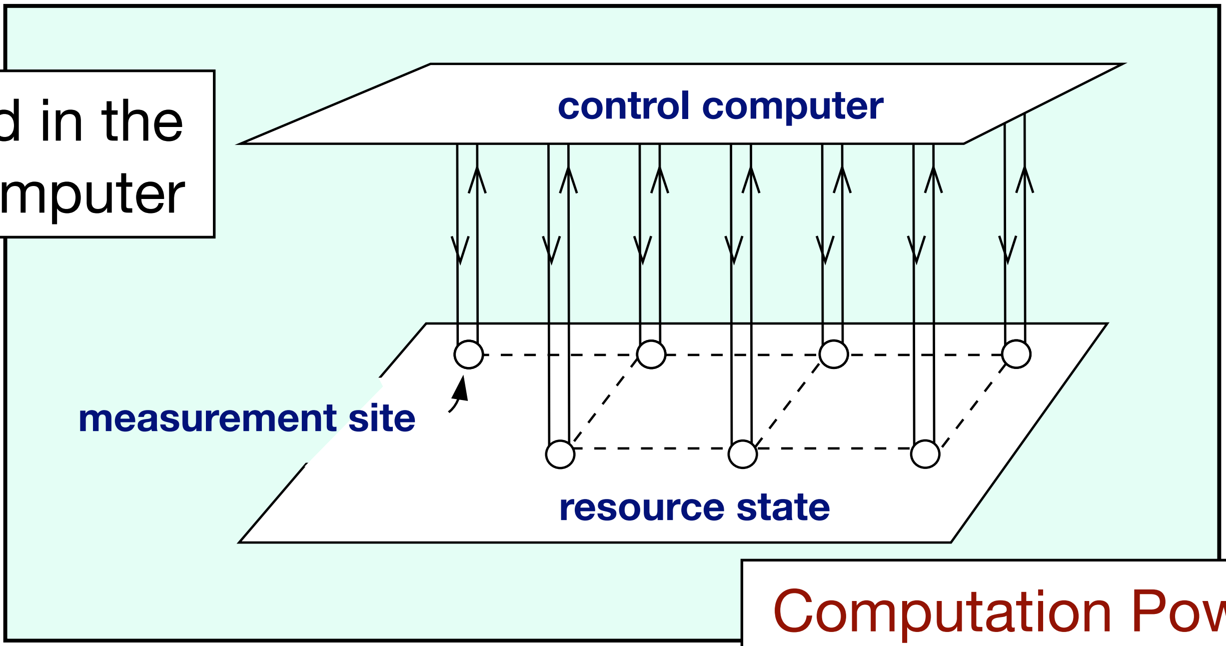
Quantum Fully Homomorphic Encryption

Quantum One-time program

Quantum Secure Multi Party Computation

Trusted Quantum Cloud Computing

Program is encoded in the classical control computer

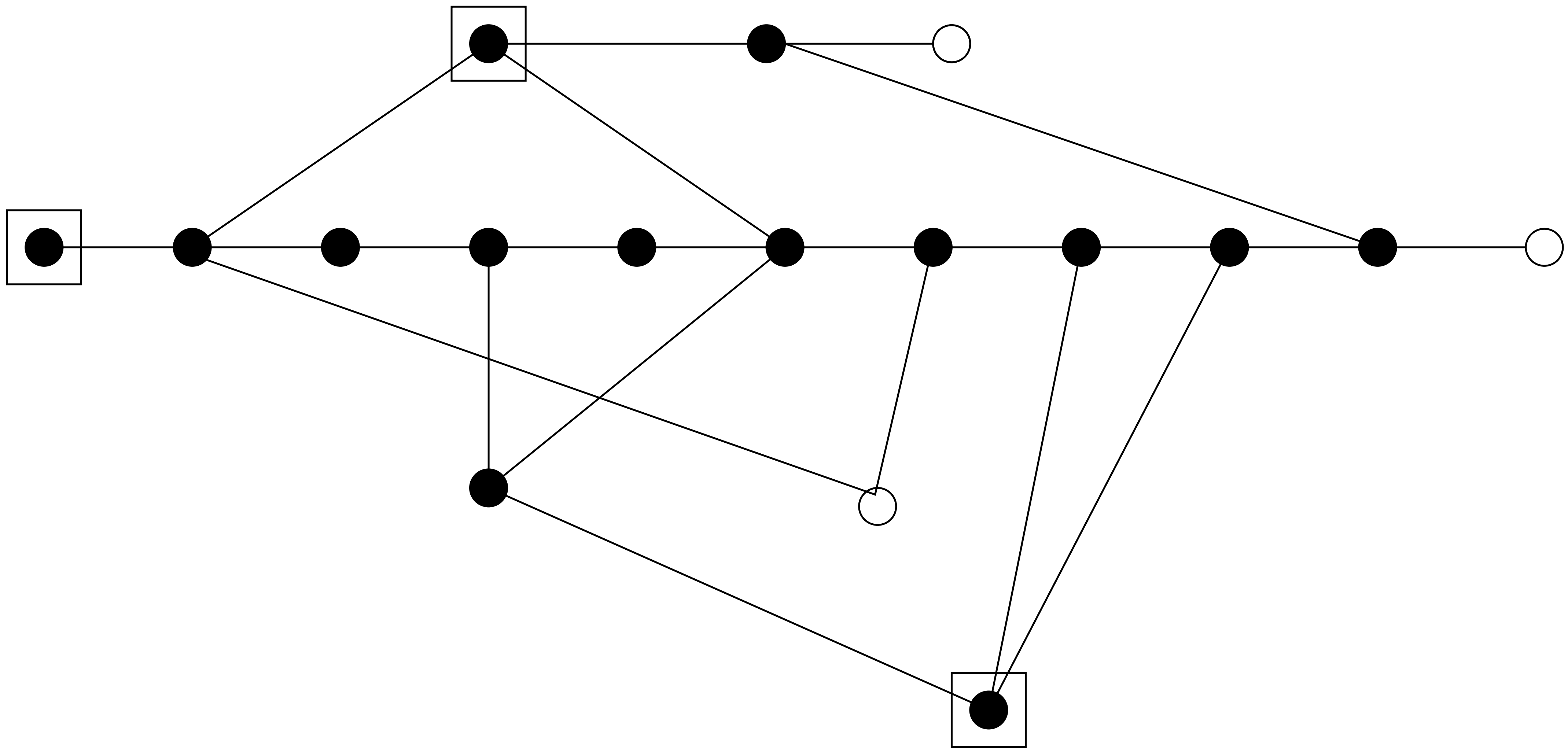


Computation Power is encoded in the quantum entanglement

Abstract Model : Measurement-based QC

- New qubits, to prepare the auxiliary qubits: N
- Entanglements, to build the quantum channel: E
- Measurements, to propagate (manipulate) qubits: M
- Corrections, to make the computation deterministic: C

Quantum Pacman



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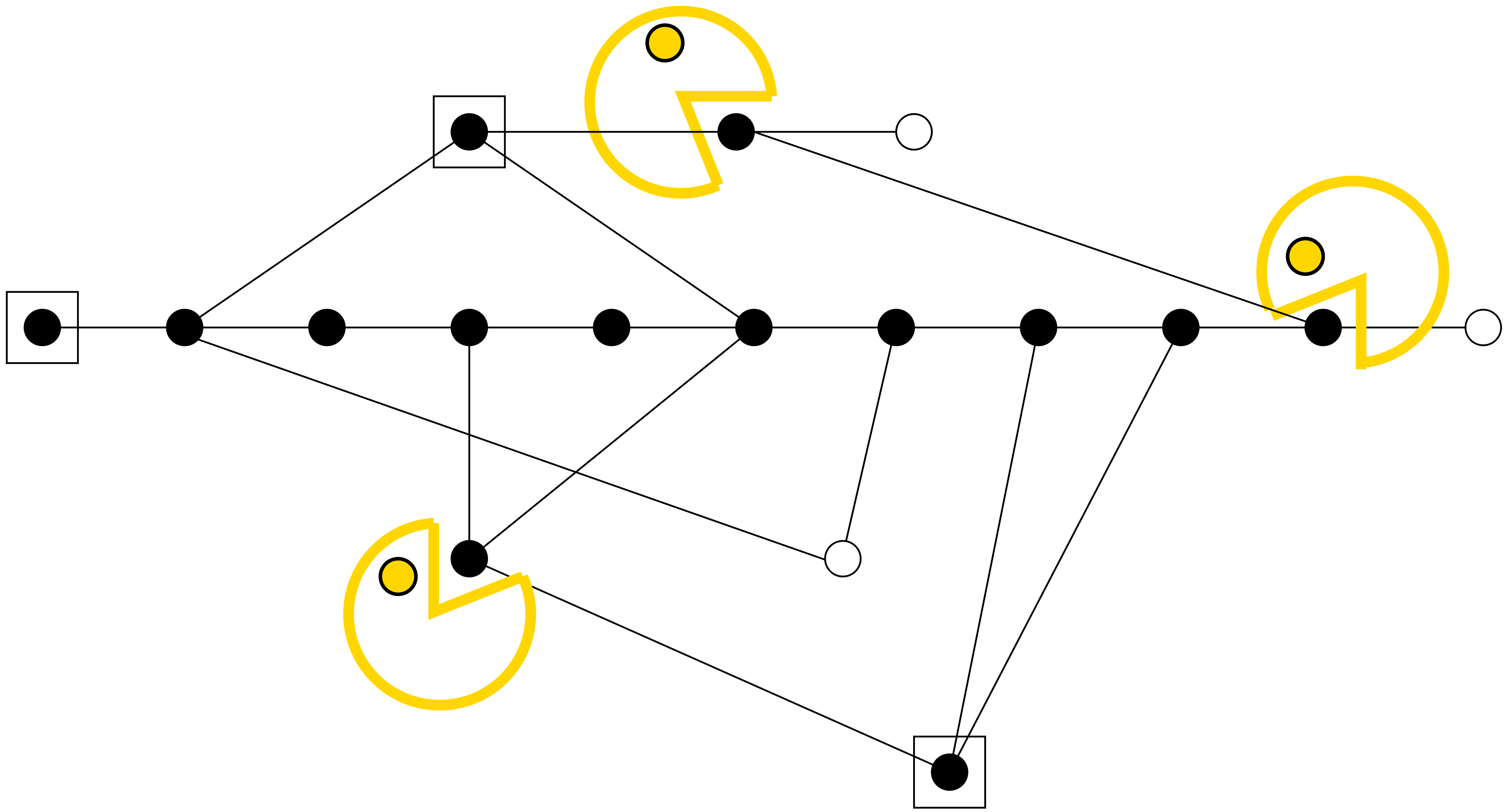

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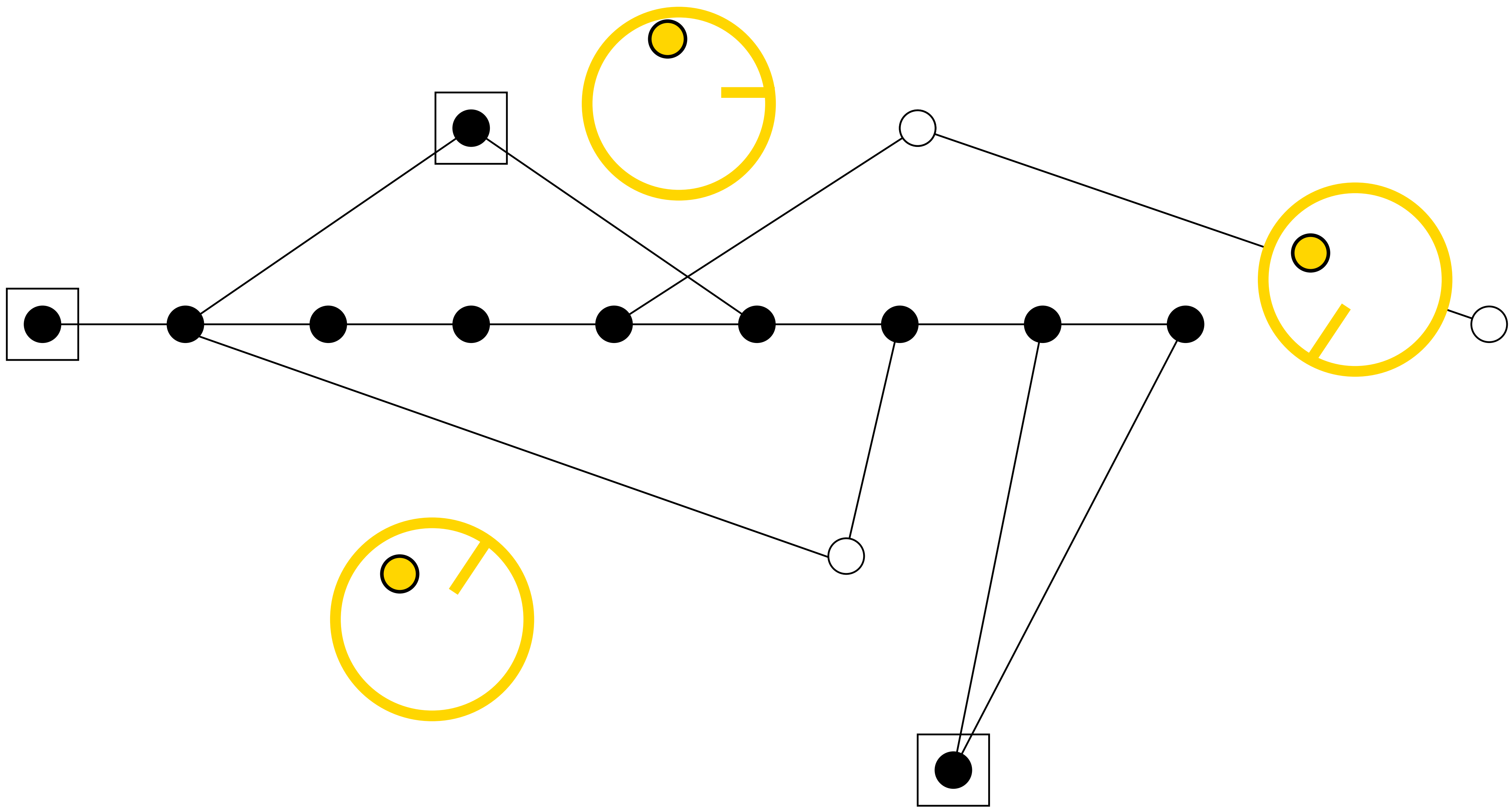

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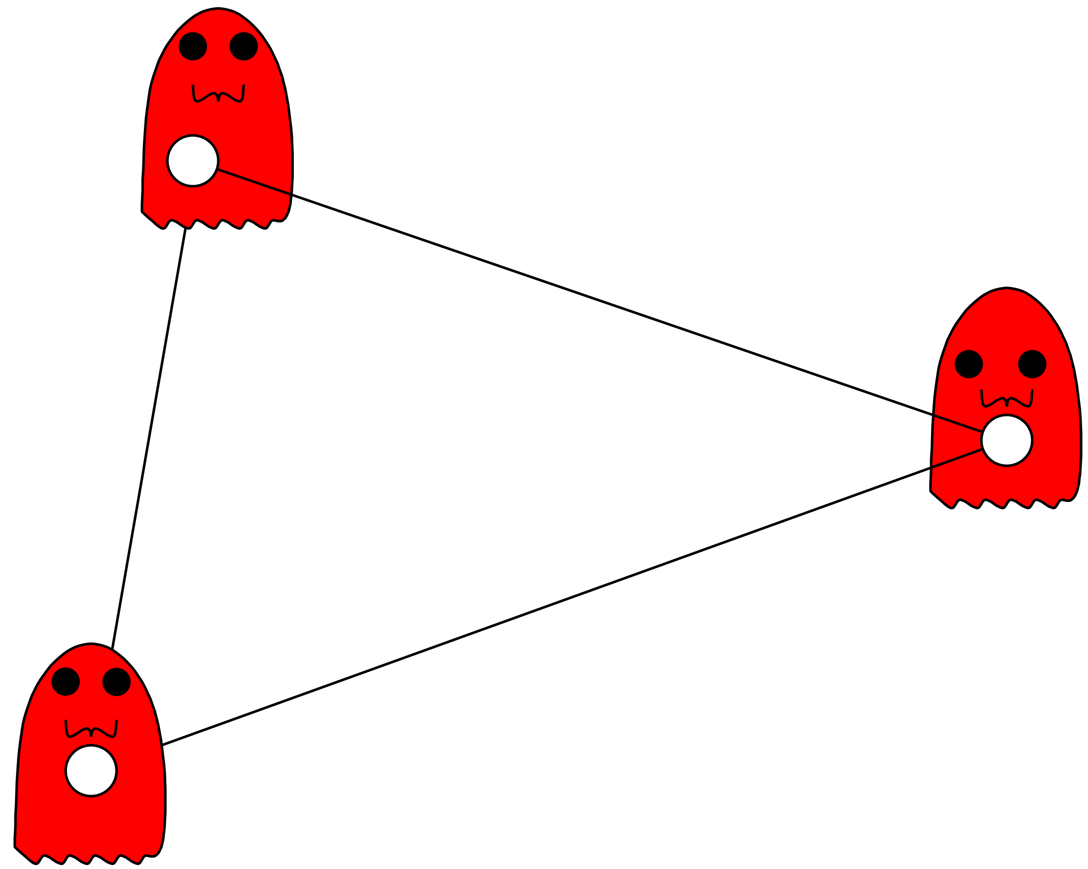
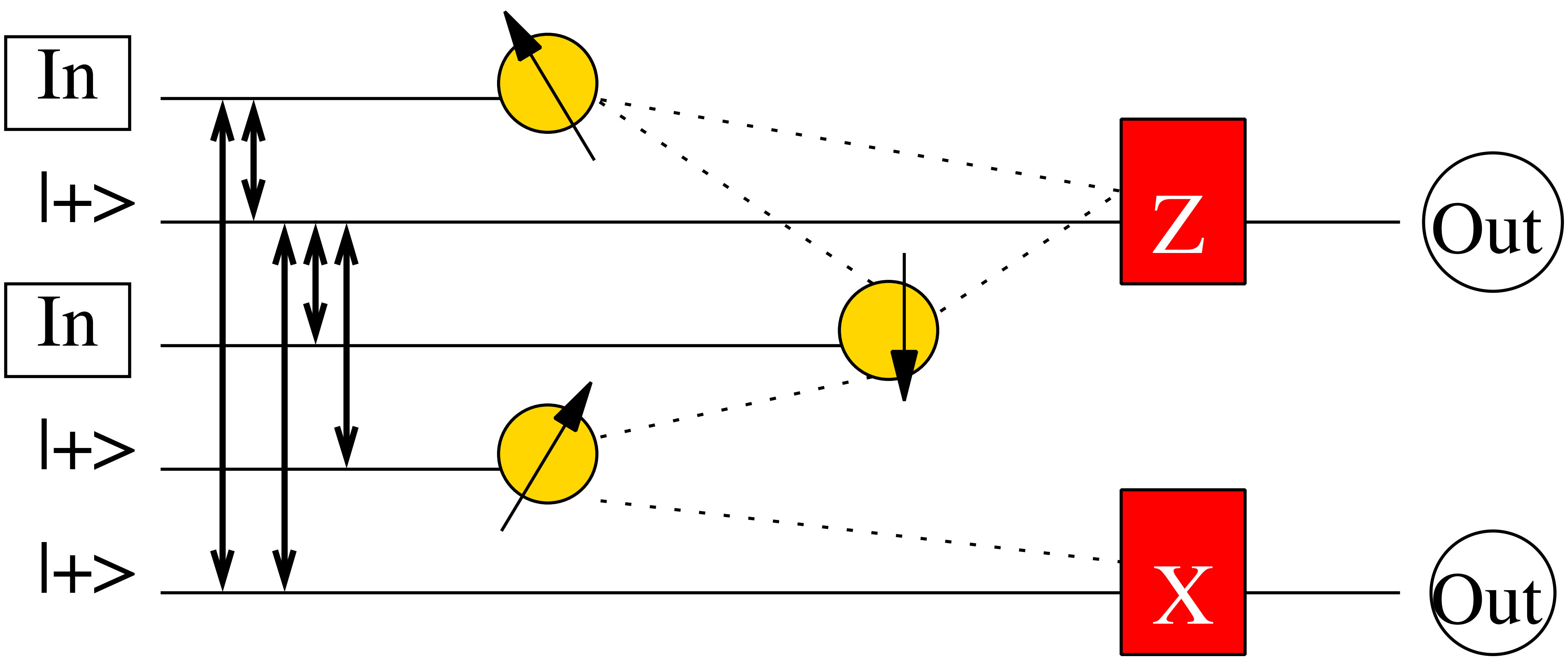
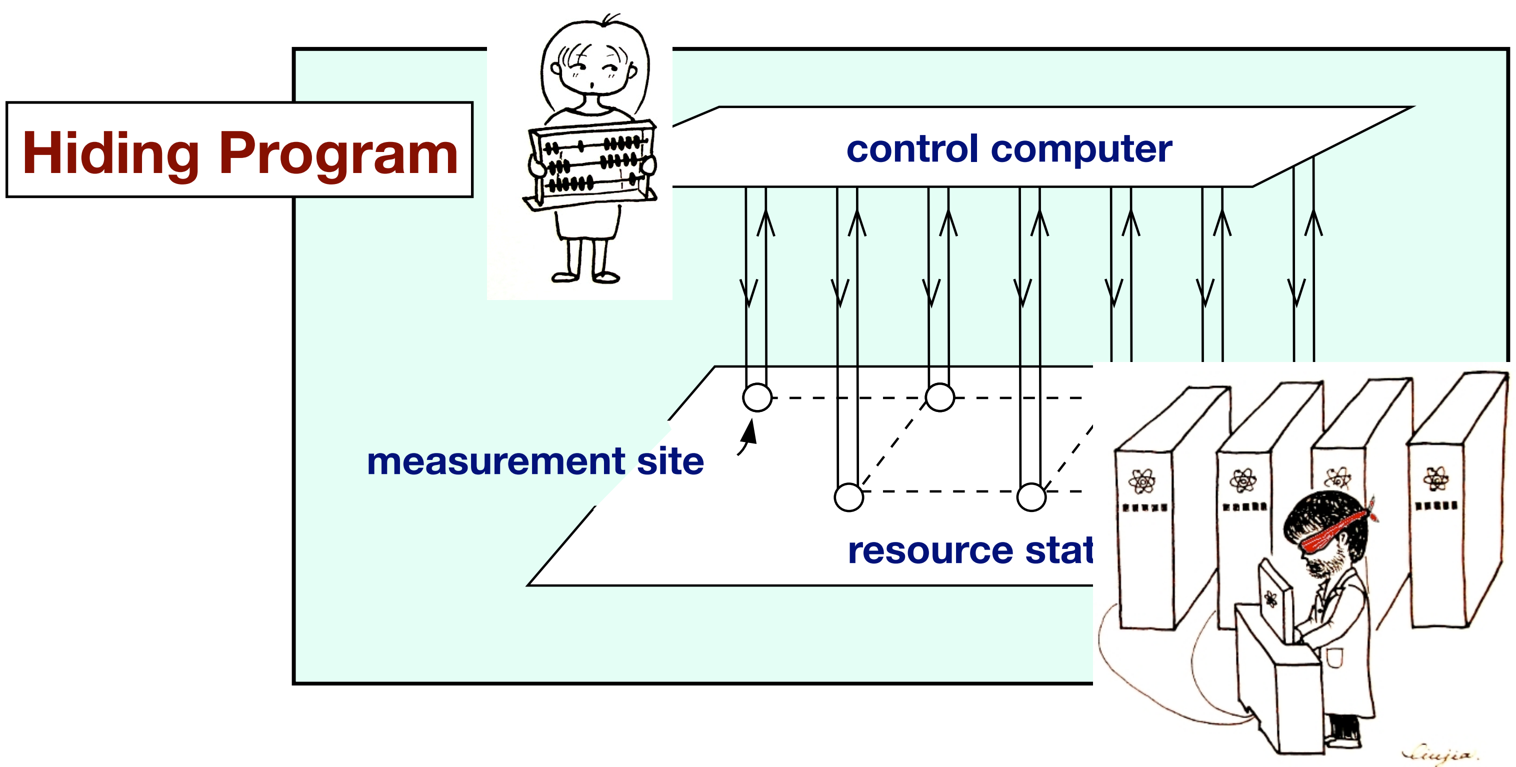


Circuit Picture



Untrusted Quantum Cloud Computing



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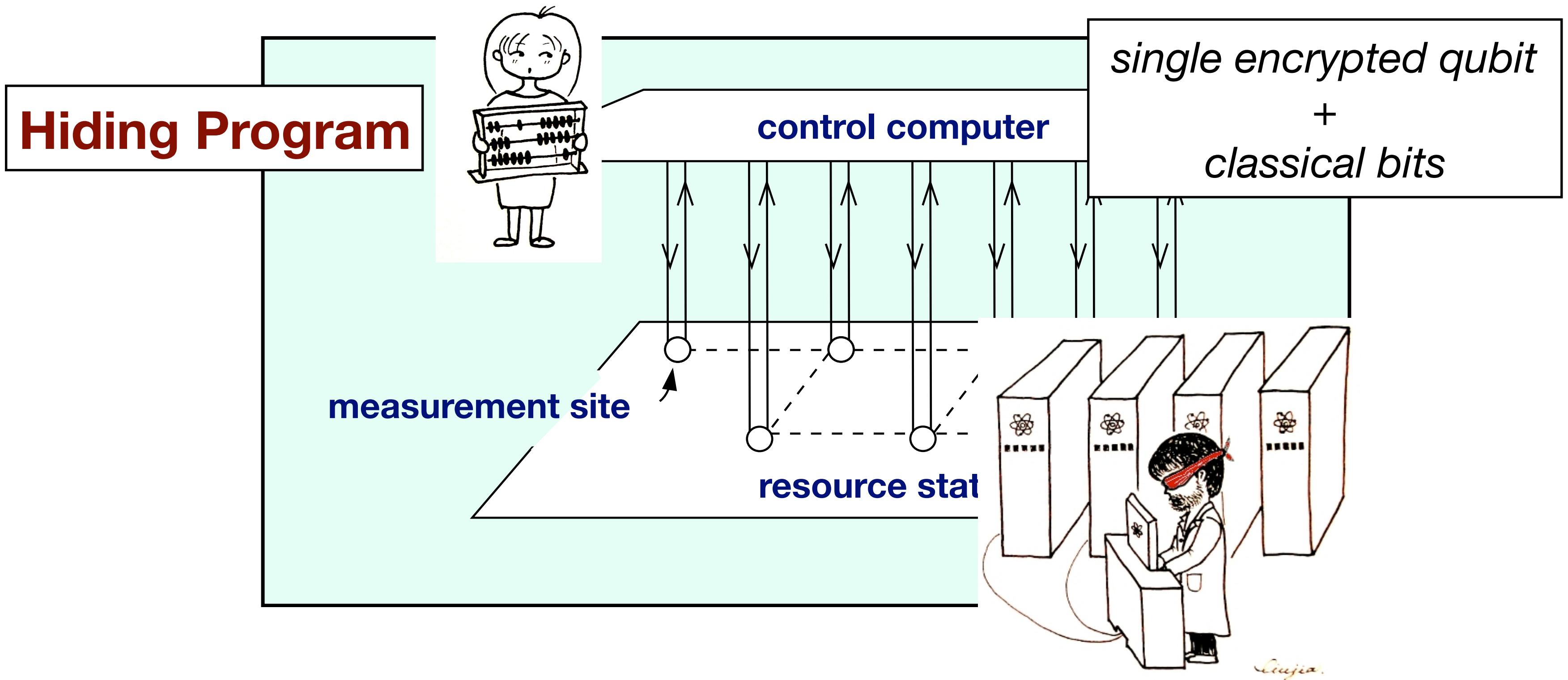

Untrusted Quantum Cloud Computing

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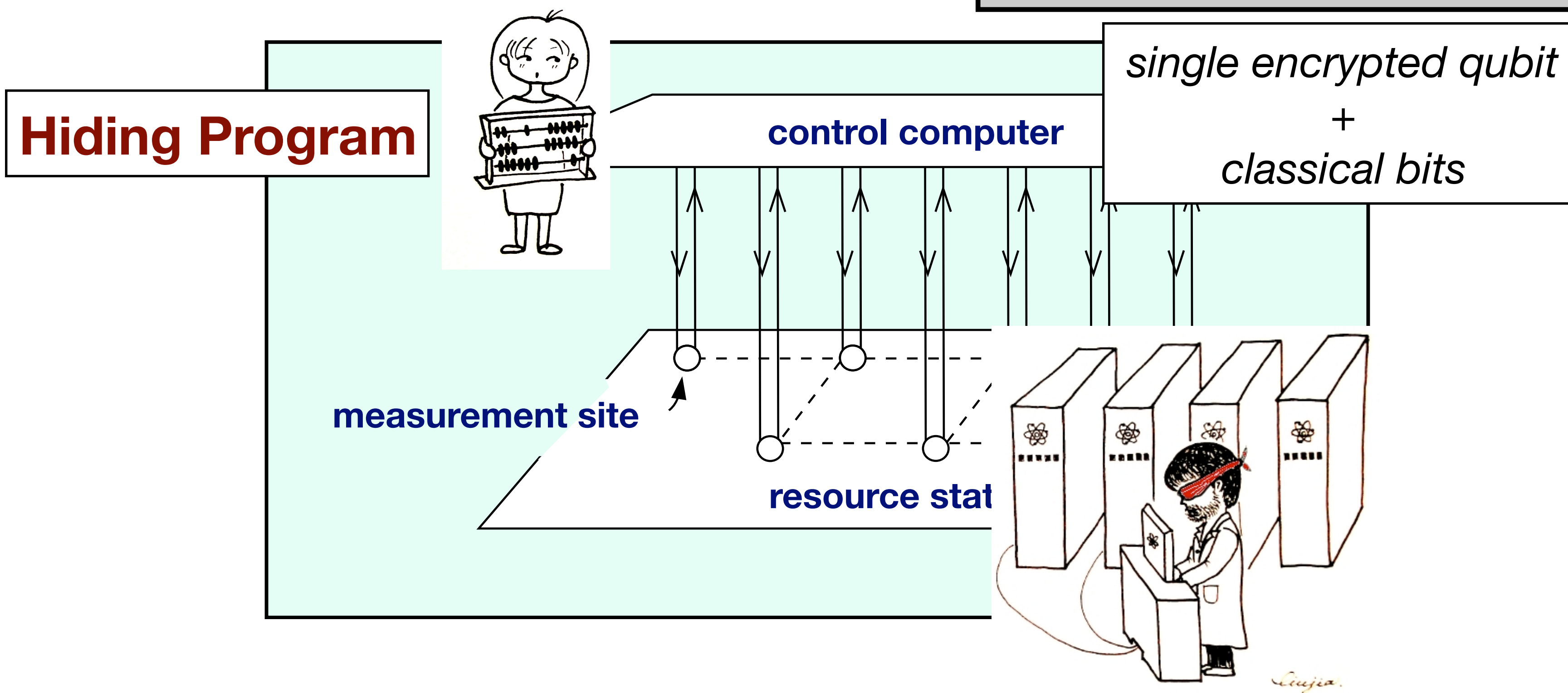

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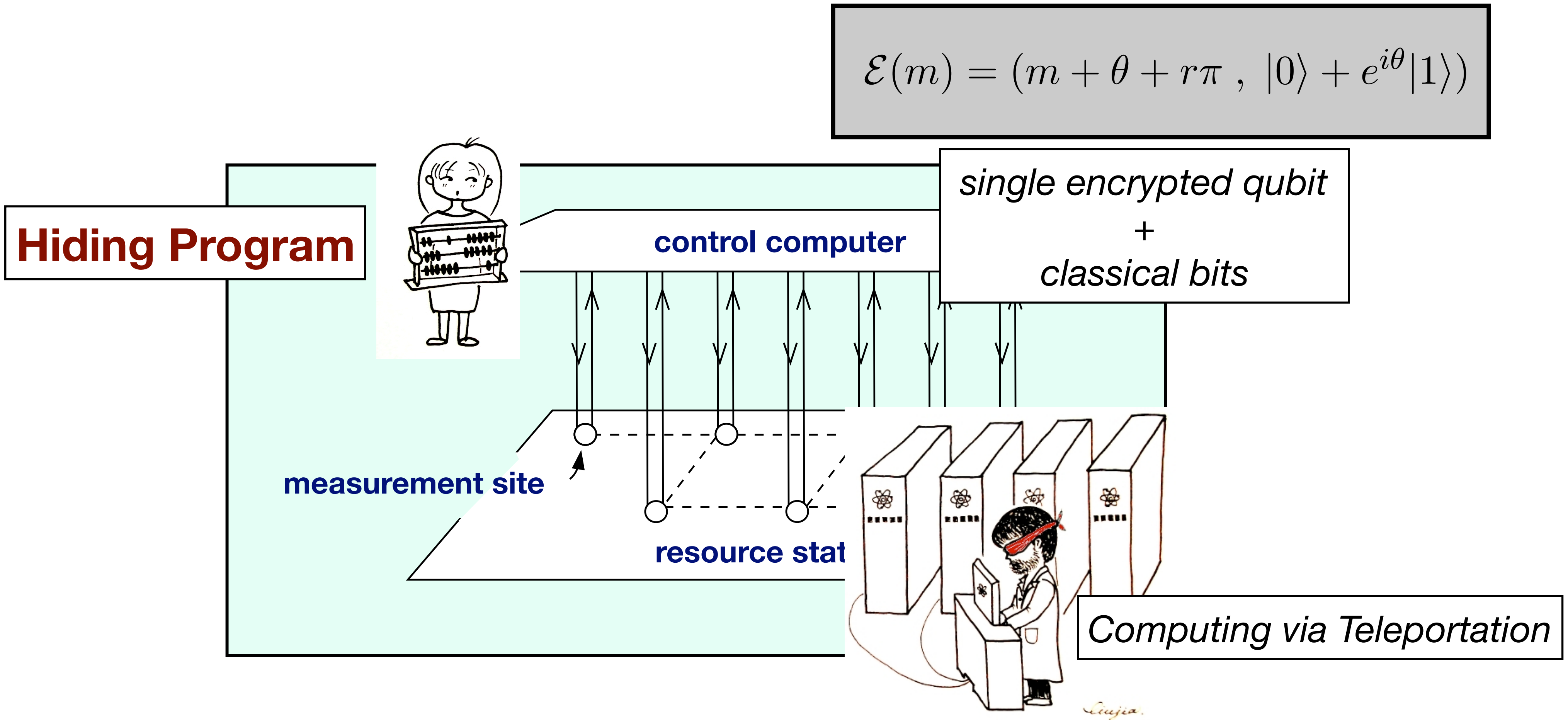

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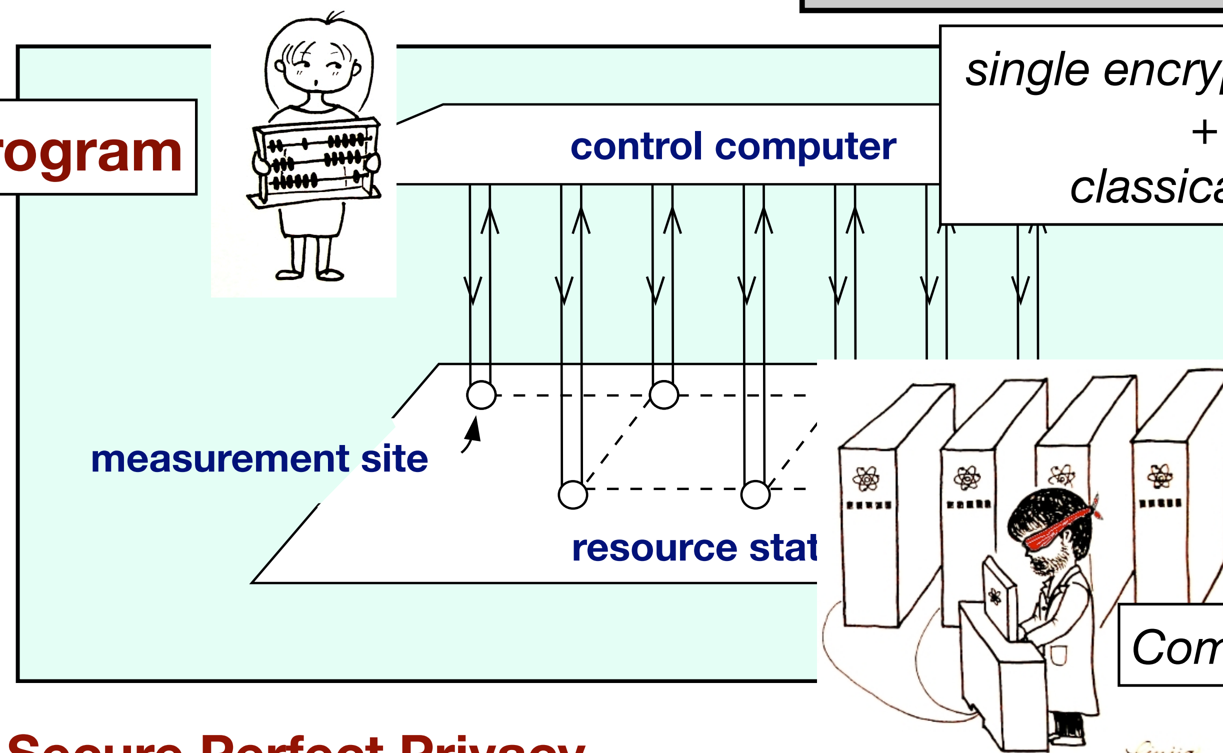
Hiding Program



single encrypted qubit
+
classical bits

Computing via Teleportation

Informationally Secure Perfect Privacy

Server learns nothing about client’s input/output/function

Hiding One Gate

$$J(\alpha) \;:=\; \frac{1}{\sqrt{2}}\begin{pmatrix}1 & e^{i\alpha}\\ 1 & -e^{i\alpha}\end{pmatrix}$$

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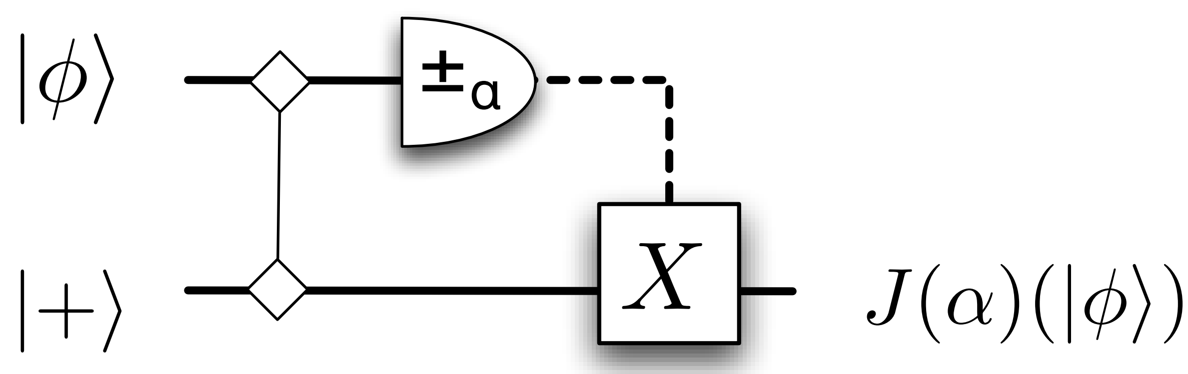

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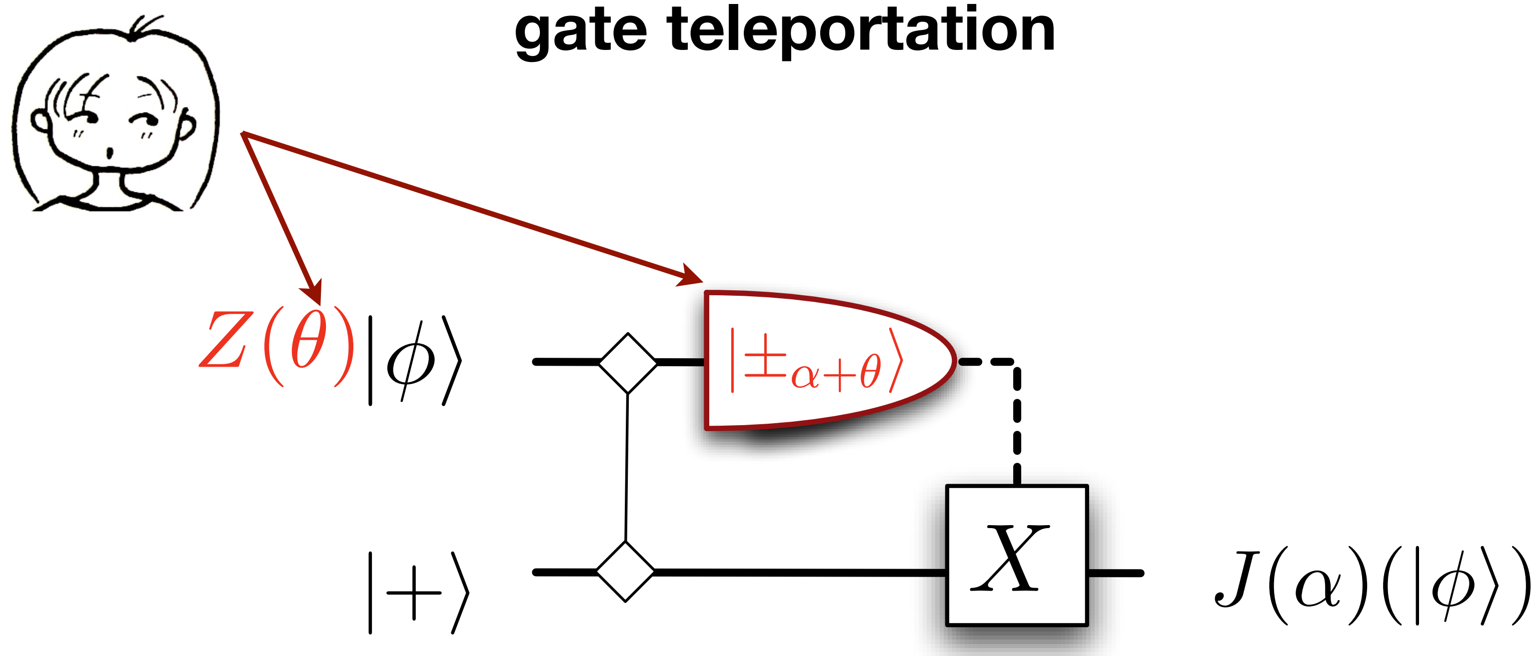

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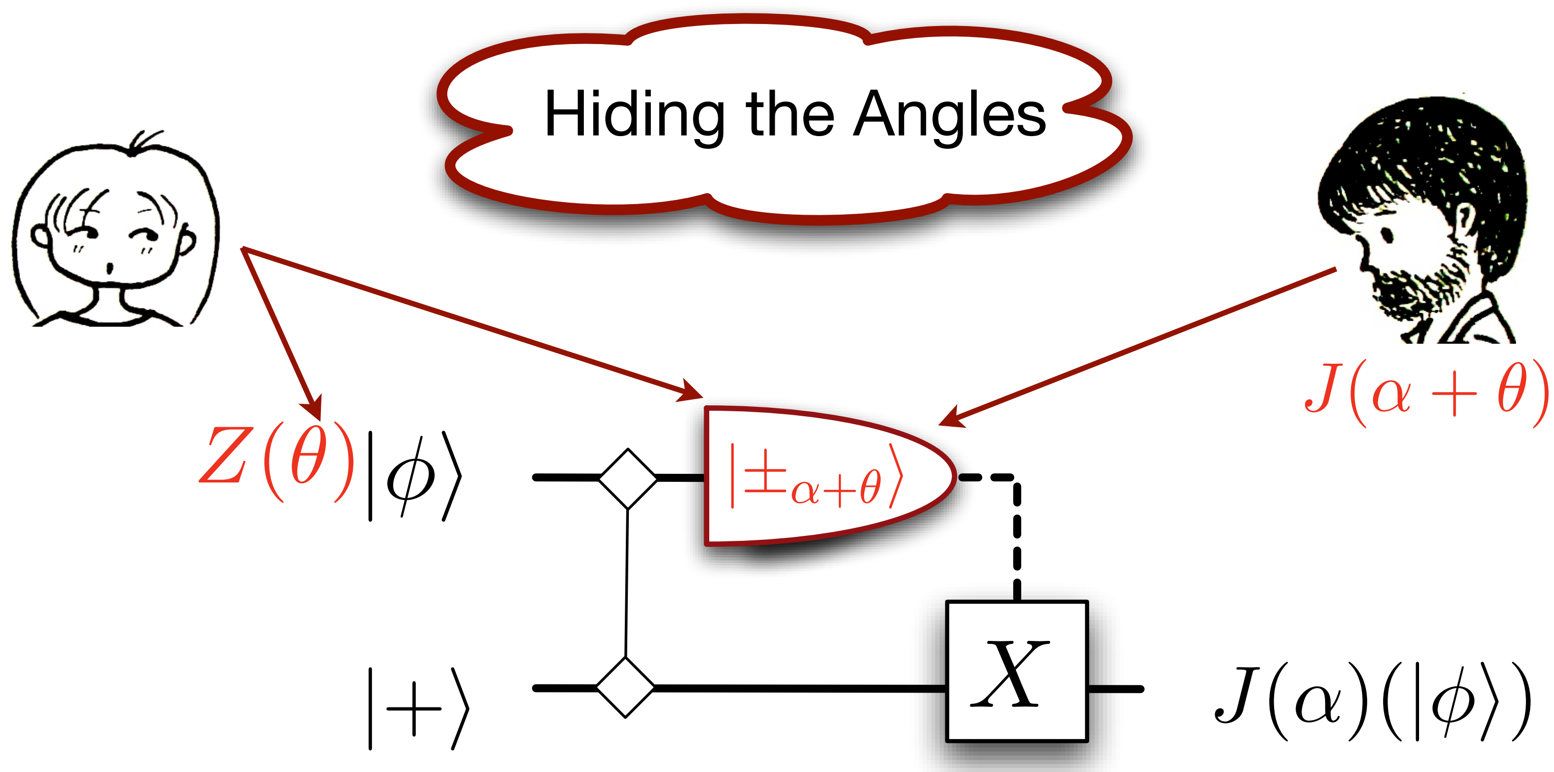

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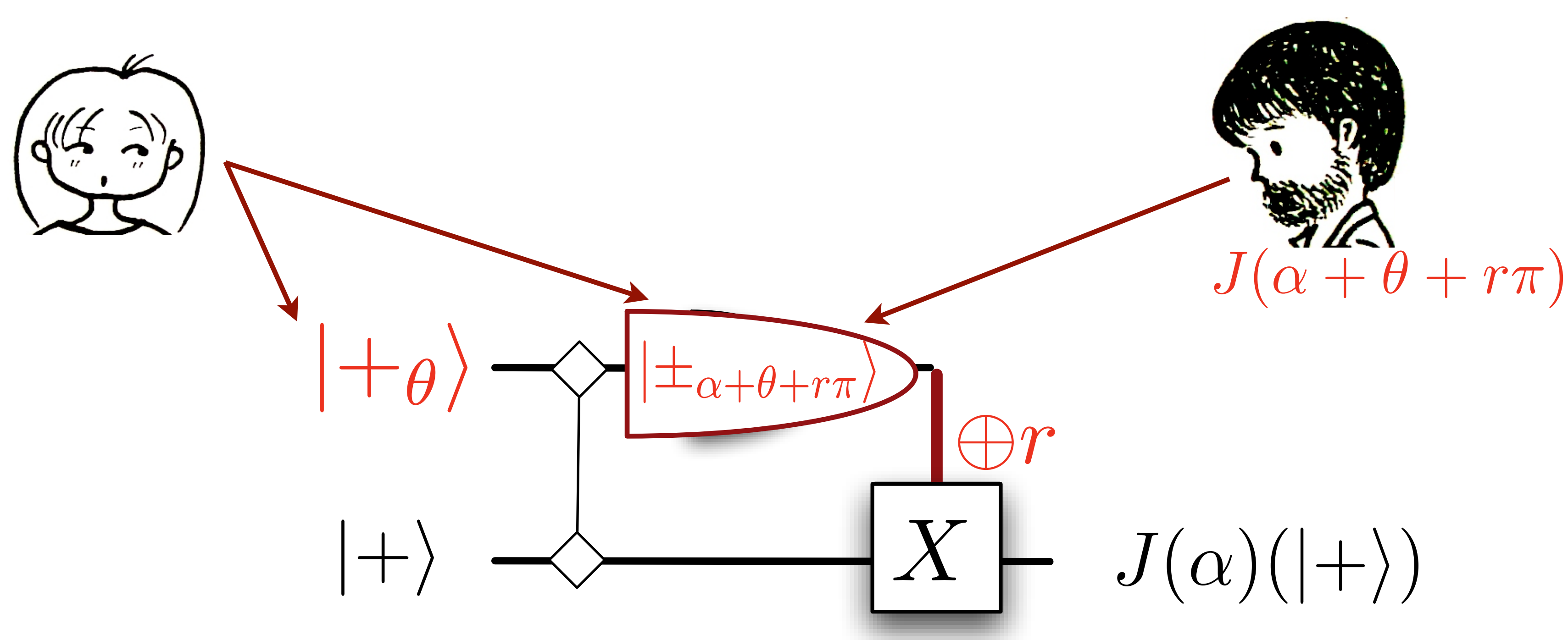
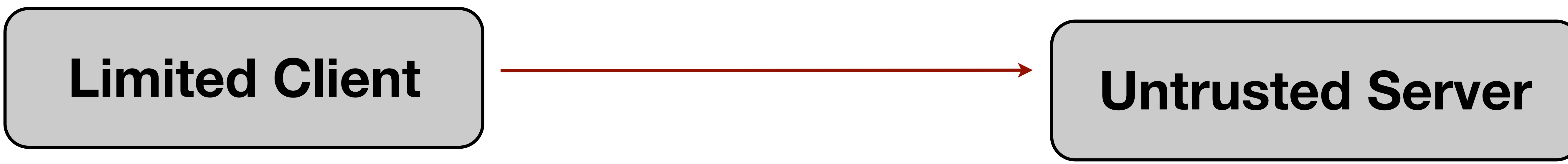


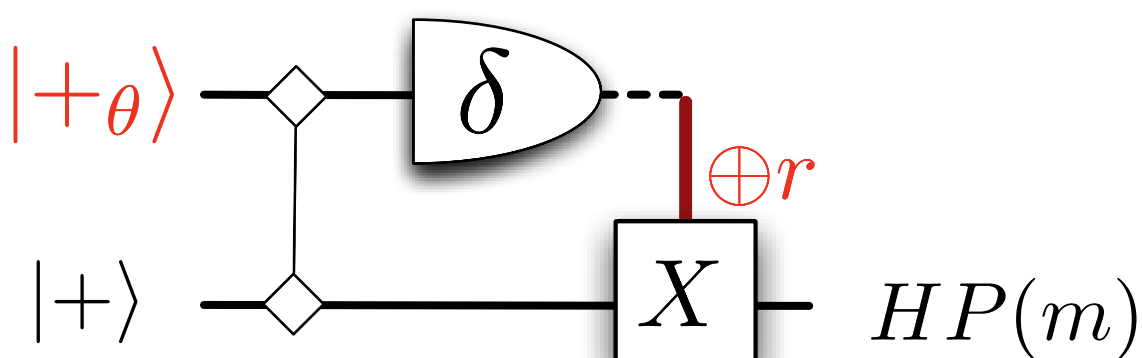
Hiding the measurement result

Hiding One Gate

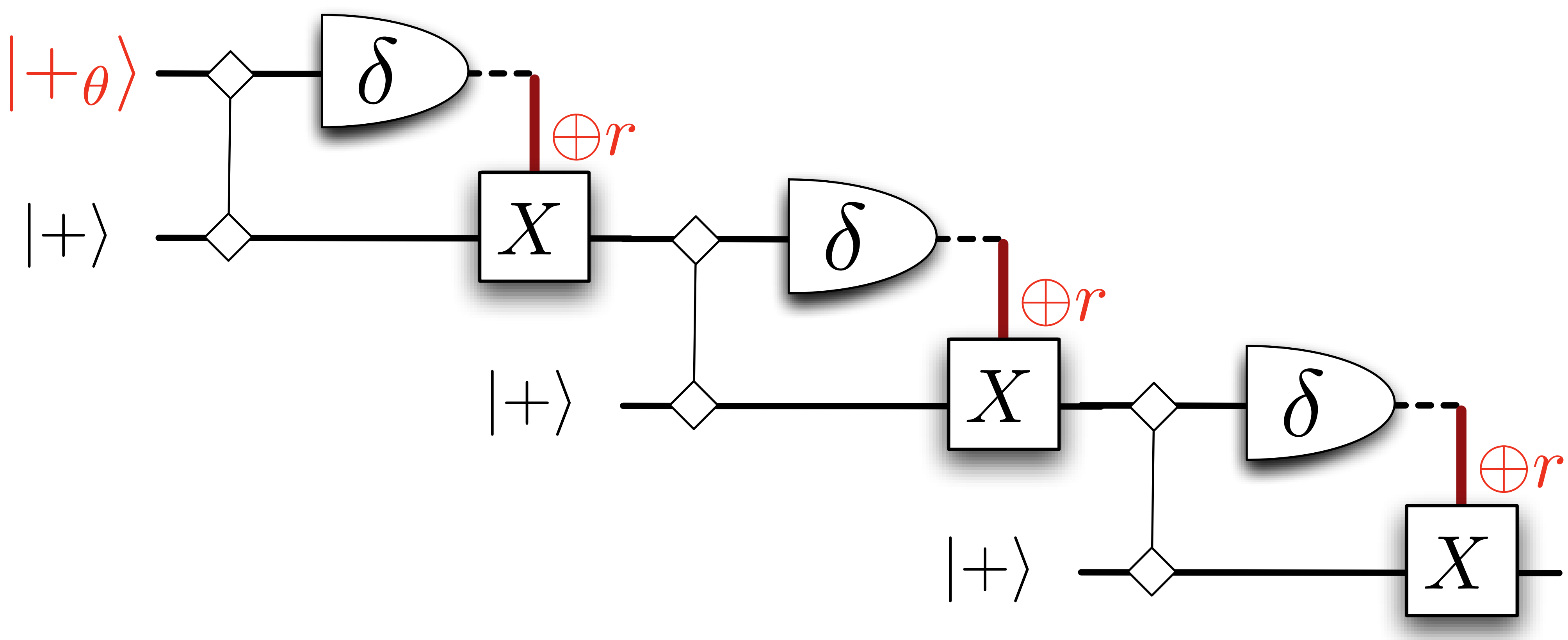
$$\theta \in_R \{0, \pi/4, 2\pi/4, \cdots, 7\pi/4\}$$

$$\mathcal{E}(m) := (\delta = m + \theta + r\pi \ , \ |+_\theta\rangle = |0\rangle + e^{i\theta}|1\rangle)$$





Gates Composition



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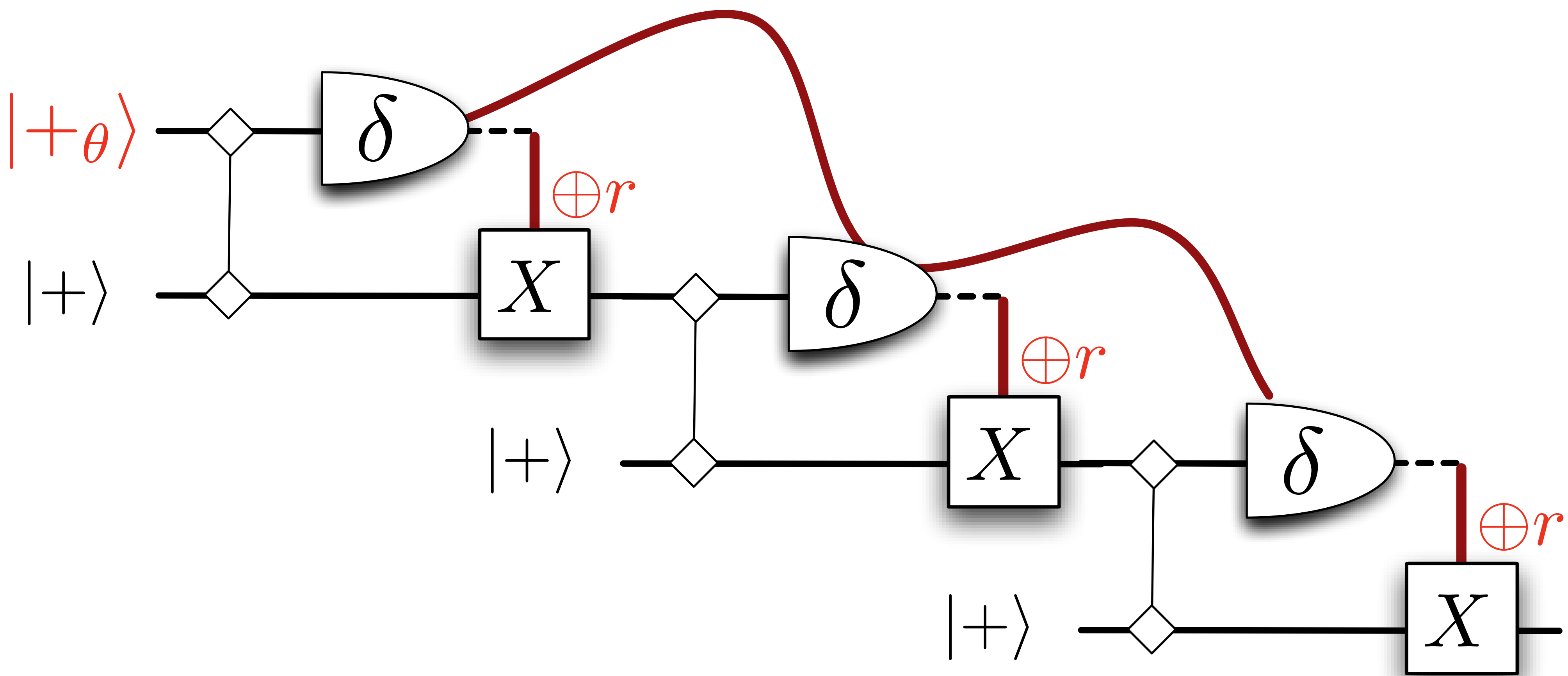
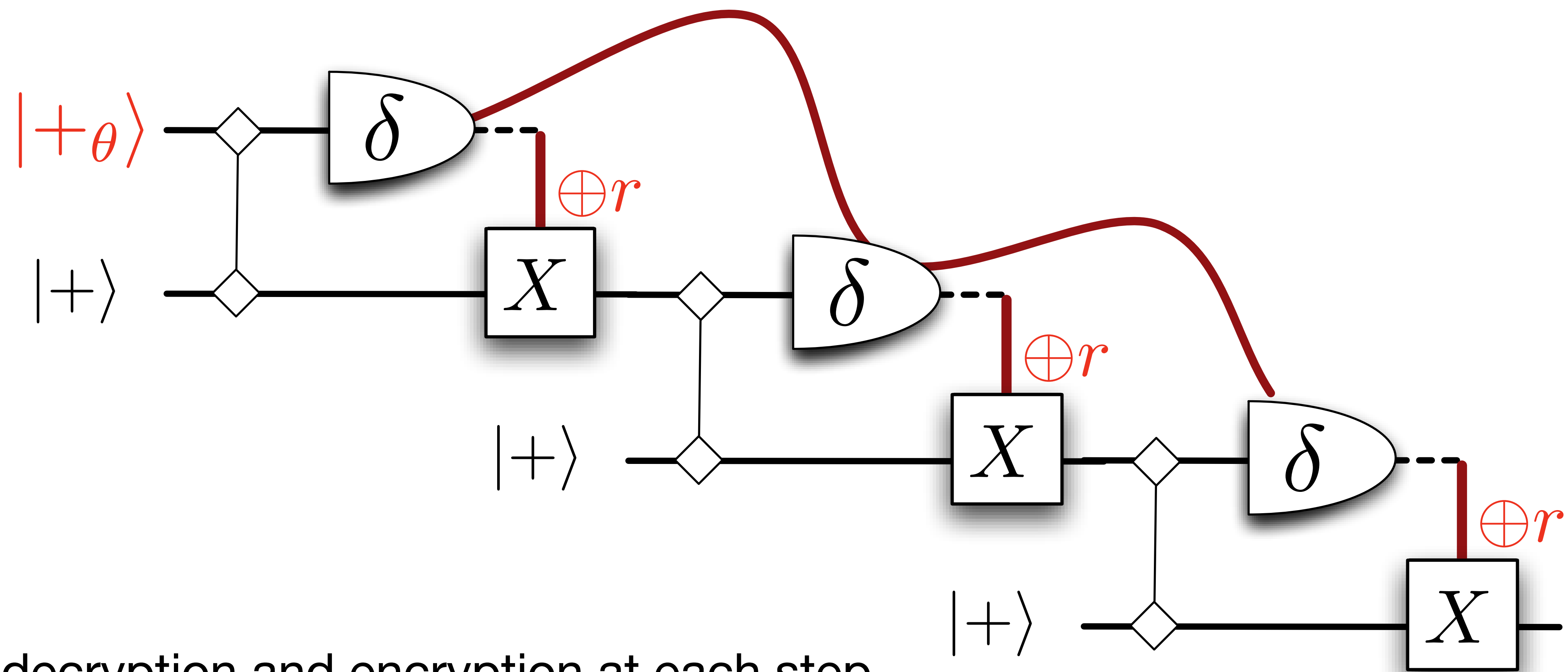


Gates Composition



Perfect decryption and encryption at each step

Client-Server interactions

Universal Blind Quantum Computings

$$X = (\tilde{U}, \{\phi_{x,y}\})$$

Universal Blind Quantum Computings

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random single qubit generator

$$1/\sqrt{2}\left(|0\rangle + e^{i\theta}|1\rangle\right)$$

$$\theta = 0, \pi/4, 2\pi/4, \ldots, 7\pi/4$$

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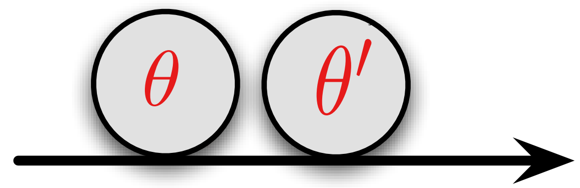

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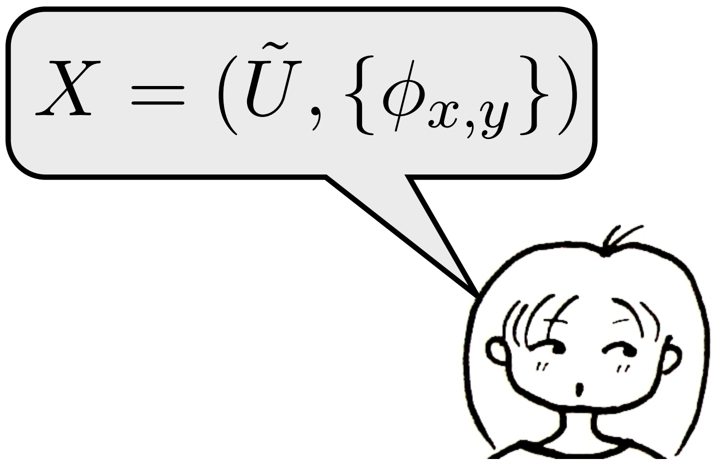

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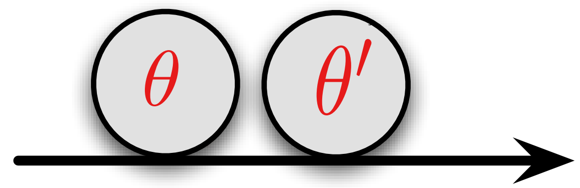

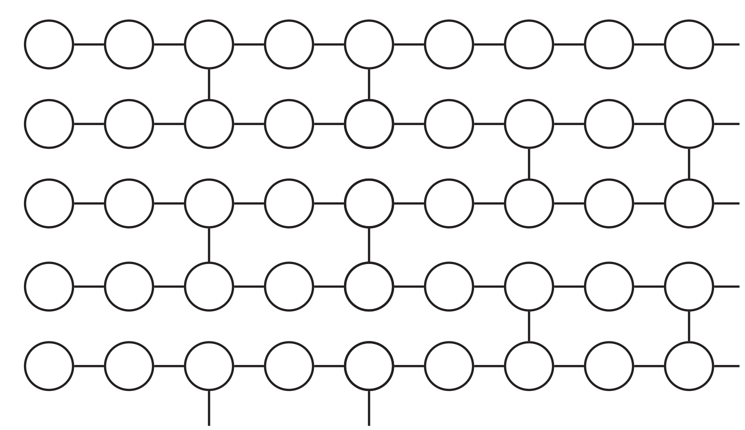

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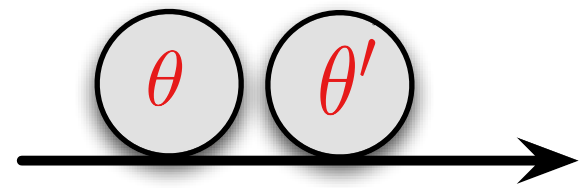

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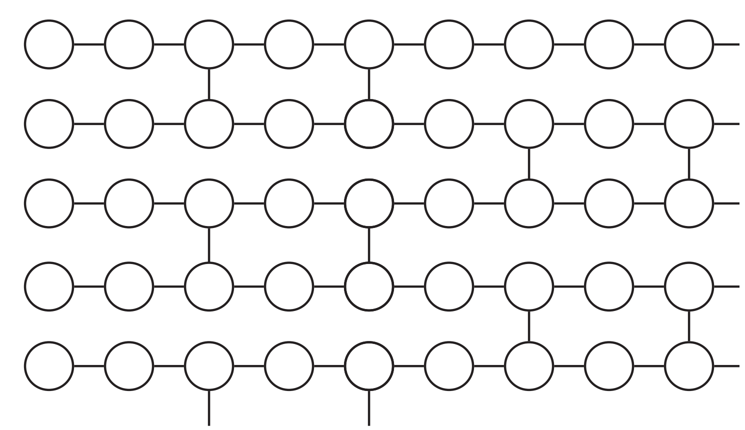

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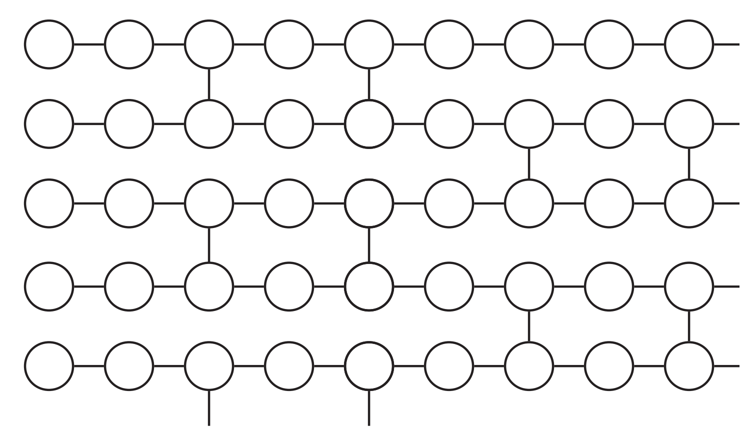

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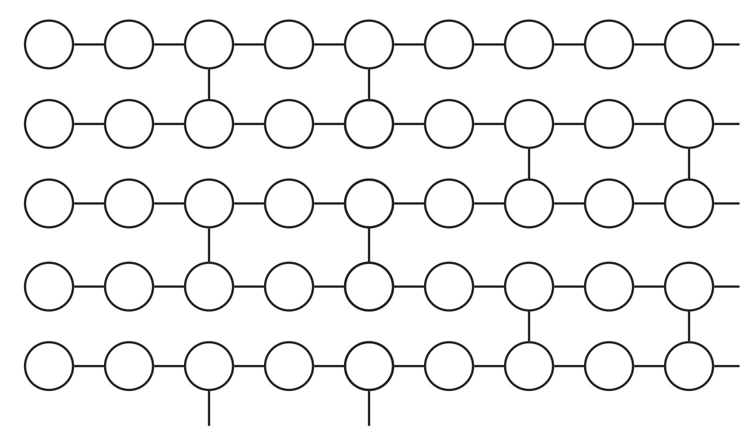

$\left\{\left|+_{\delta_{x,y}}\right\rangle, \left|-_{\delta_{x,y}}\right\rangle\right\}$

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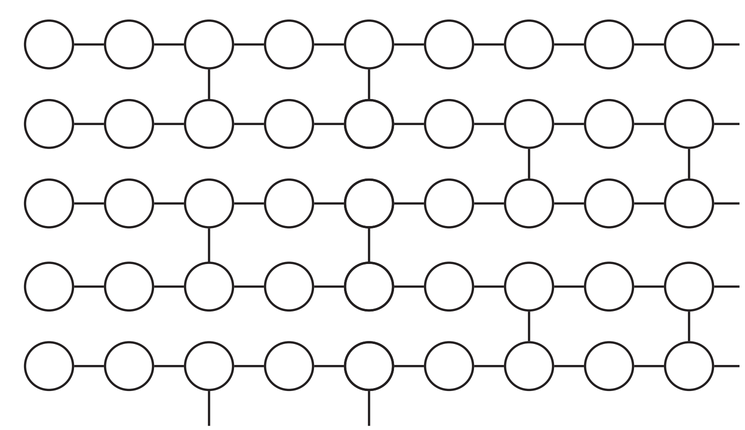

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$s_{x,y} \in \{0,1\}$

$$\{\left|+_{\delta_{x,y}}\right\rangle, \left|-_{\delta_{x,y}}\right\rangle\}$$

Universal Blind Quantum Computings

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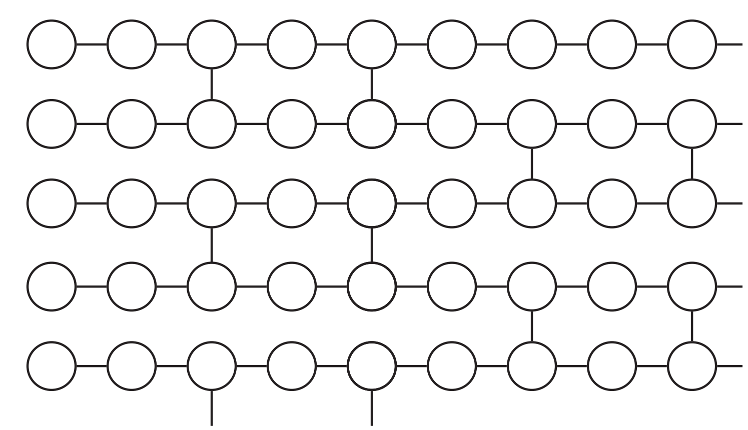

$$\left\{\left|+_{\delta_{x,y}}\right\rangle, \left|-_{\delta_{x,y}}\right\rangle\right\}$$

$s_{x,y} \in \{0,1\}$

$$s_{x,y} := s_{x,y} + r_{x,y}$$

Blindness

Protocol P on input $X = (\tilde{U}, \{\phi_{x,y}\})$ leaks at most $L(X)$

➡ The distribution of the classical information obtained by Bob is independent of X

➡ Given the above distribution, the quantum state is fixed and independent of X

Proof (L(X)=m,n)

- Independence of Bob’s classical information

Proof (L(X)=m,n)

➡ Independence of Bob's classical information

$$\theta_{x,y} \in_R \{0, \cdots, 7\pi/4\}$$

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➡ Independence of Bob's quantum information for a fixed δ

Proof (L(X)=m,n)

➡ Independence of Bob's classical information

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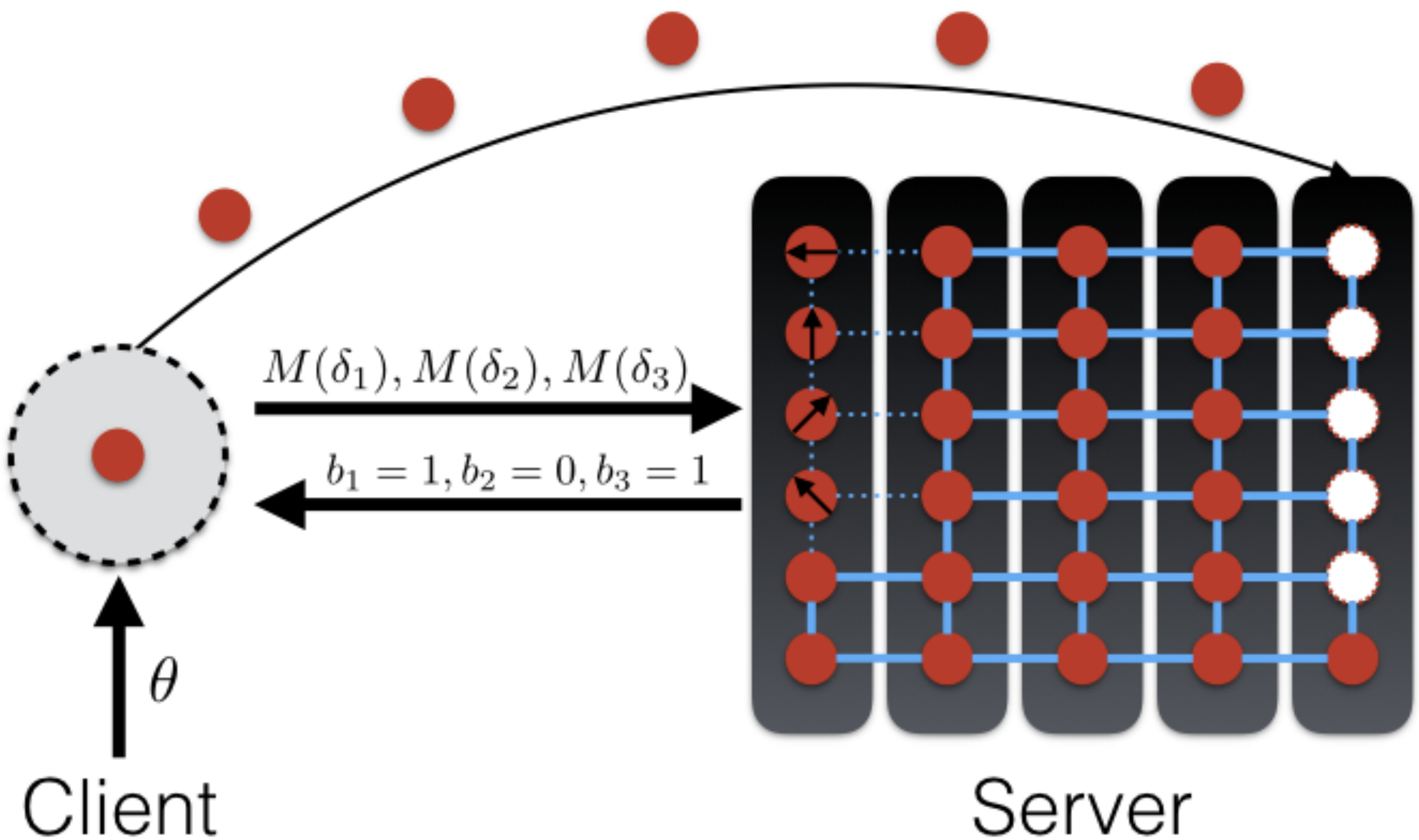
$$\delta_{x,y} = \phi'_{x,y} + \theta_{x,y} + \pi r_{x,y}$$

➡ Independence of Bob's quantum information for a fixed δ

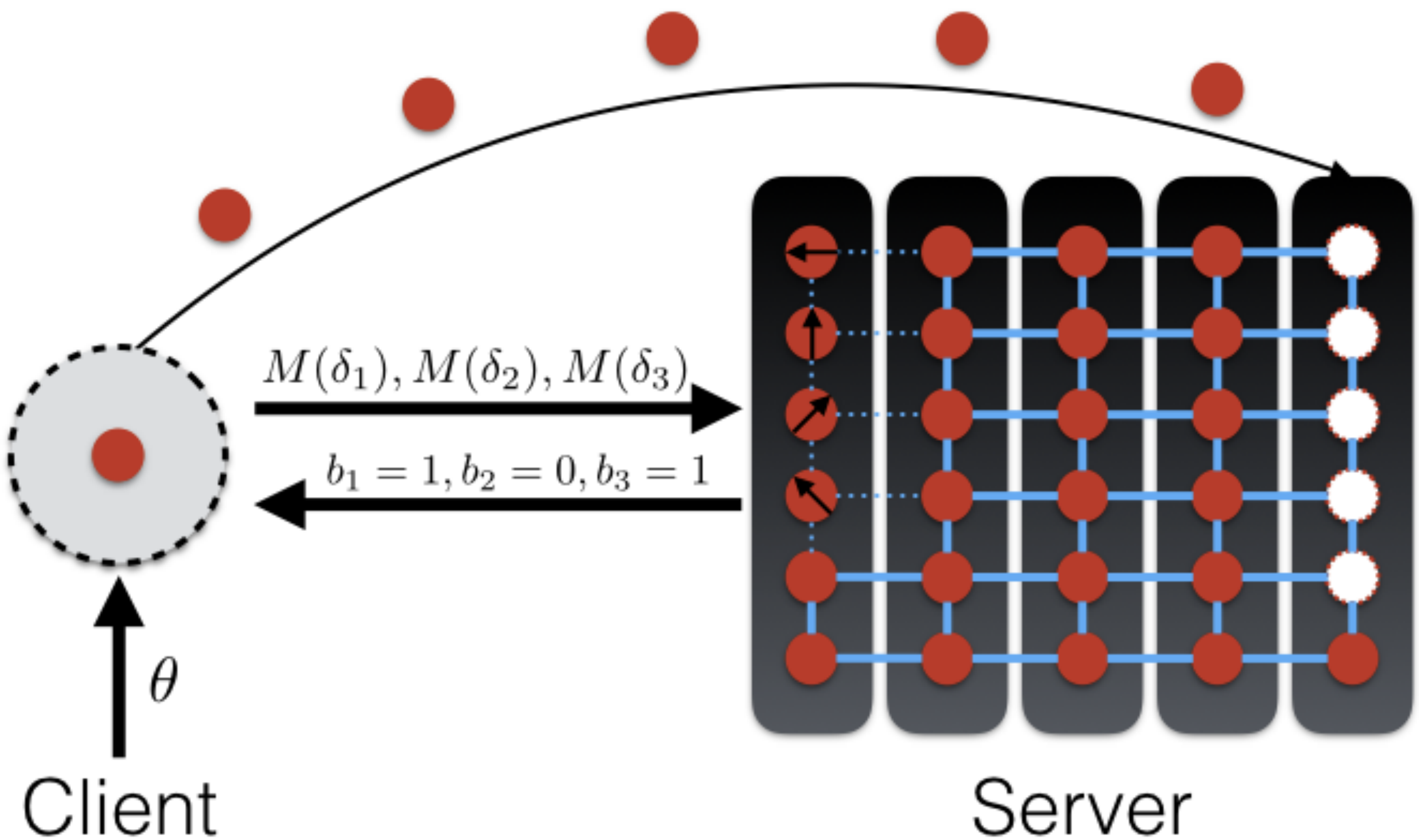
1. $r_{x,y} = 0$ so $\delta_{x,y} = \phi'_{x,y} + \theta'_{x,y}$ and $|\psi_{x,y}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i(\delta_{x,y} - \phi'_{x,y})}|1\rangle)$.

2. $r_{x,y} = 1$ so $\delta_{x,y} = \phi'_{x,y} + \theta'_{x,y} + \pi$ and $|\psi_{x,y}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - e^{i(\delta_{x,y} - \phi'_{x,y})}|1\rangle)$.

Informationally Secure Quantum Cloud

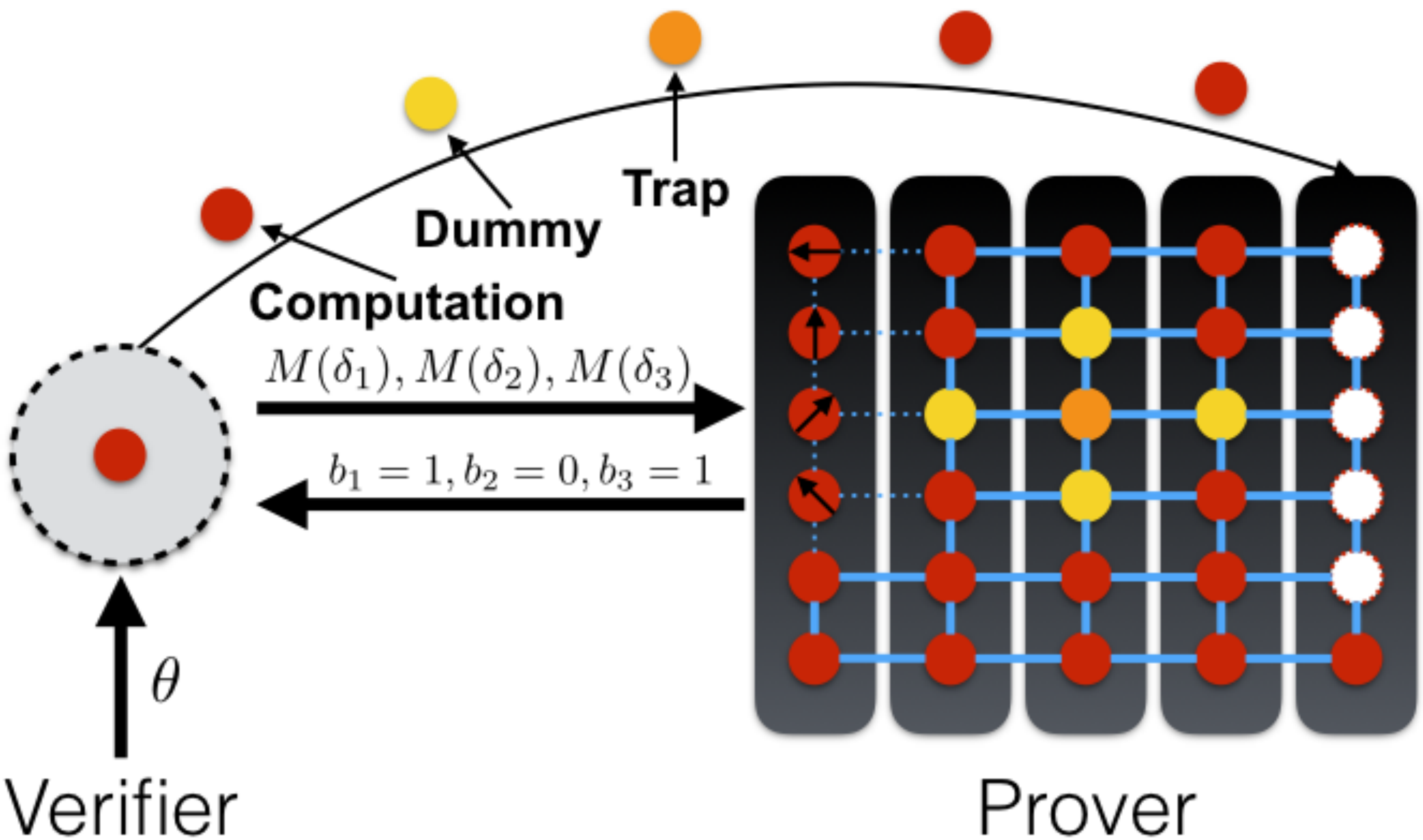


Informationally Secure Quantum Cloud

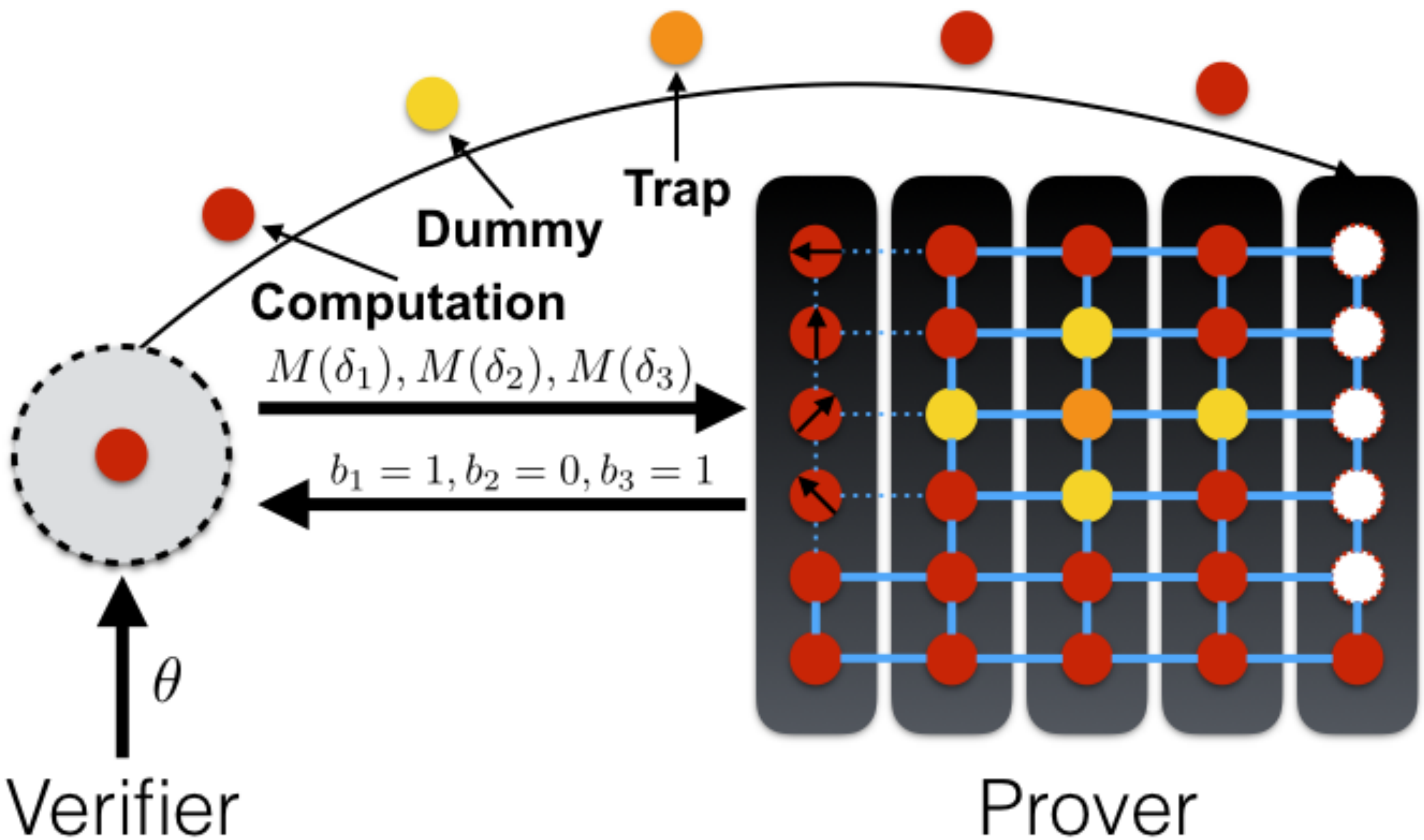


Universal Blind Quantum Computing: QKD + Teleportation

Verifiable Outsourced Computing

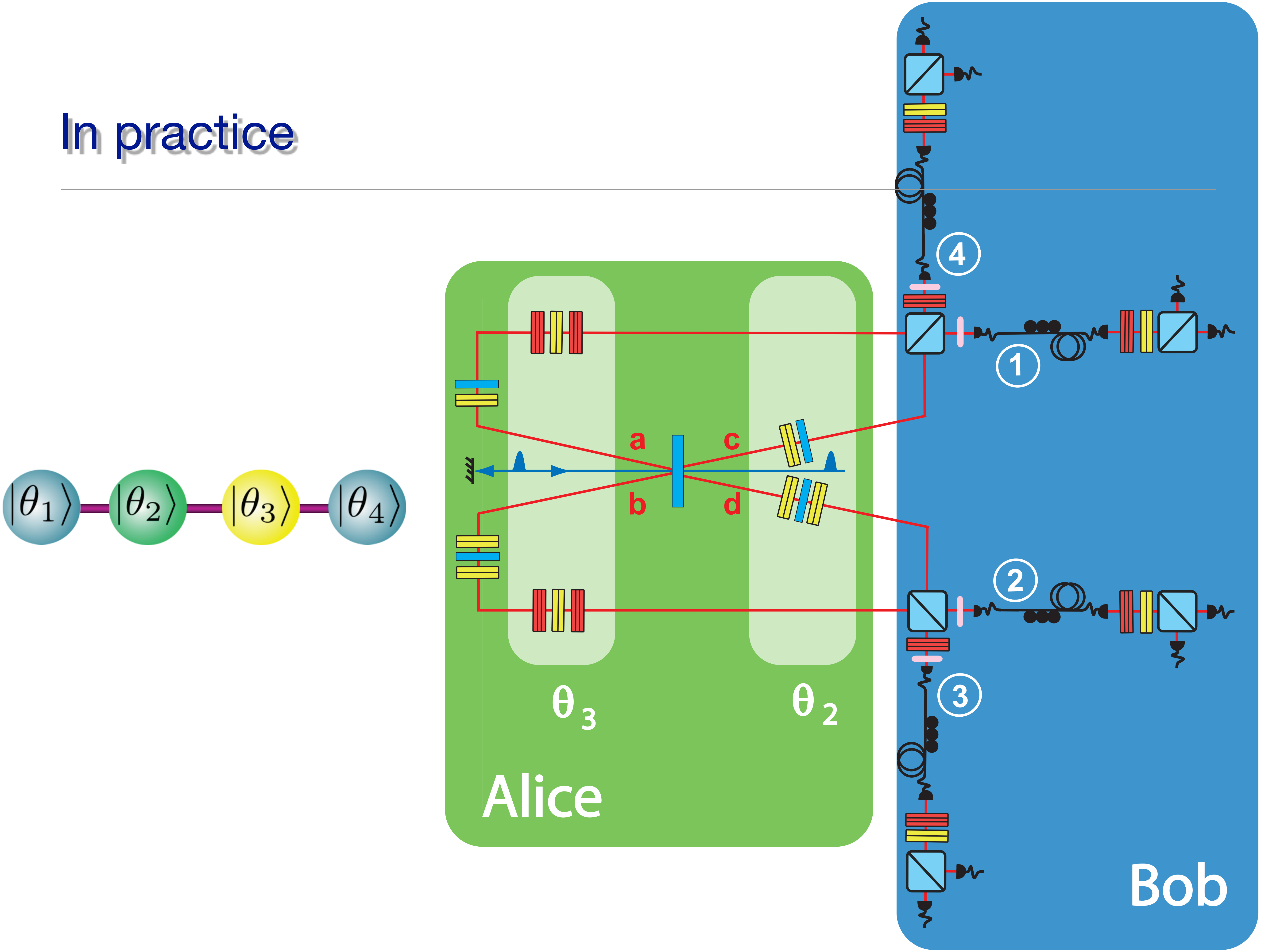


Verifiable Outsourced Computing



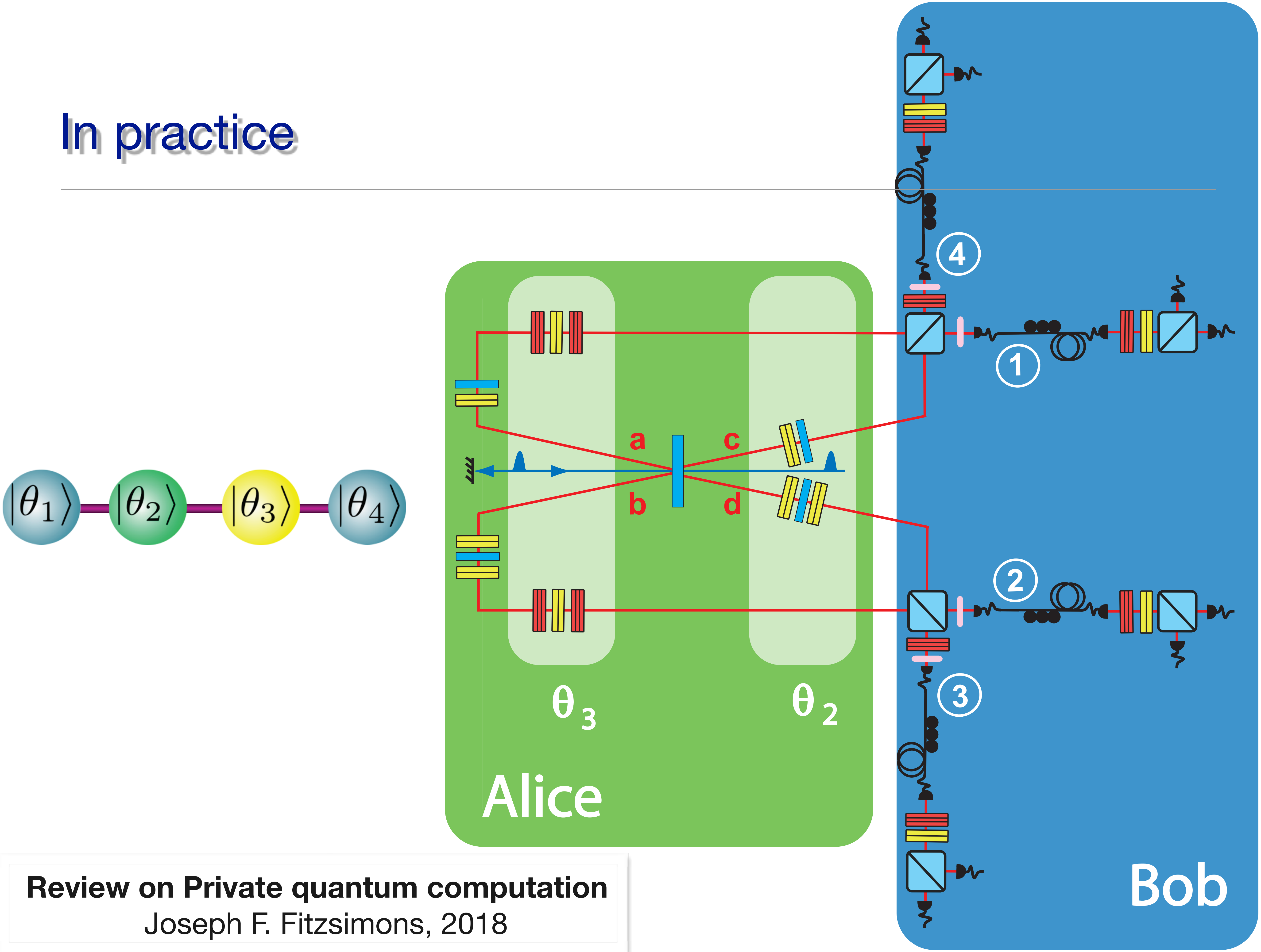
Verifiable Universal Blind Quantum Computing: QKD + Teleportation + Test

In practice



$|\theta_1\rangle$
$|\theta_2\rangle$
$|\theta_3\rangle$
$|\theta_4\rangle$
a
b
c
d
θ 3
θ 2
Alice
1
2
3
4
Bob

In practice

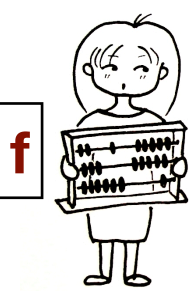


Review on Private quantum computation
Joseph F. Fitzsimons, 2018

Yao Garbled Circuit - Secure 2-party Computing

Secret input a

Garbled Program f



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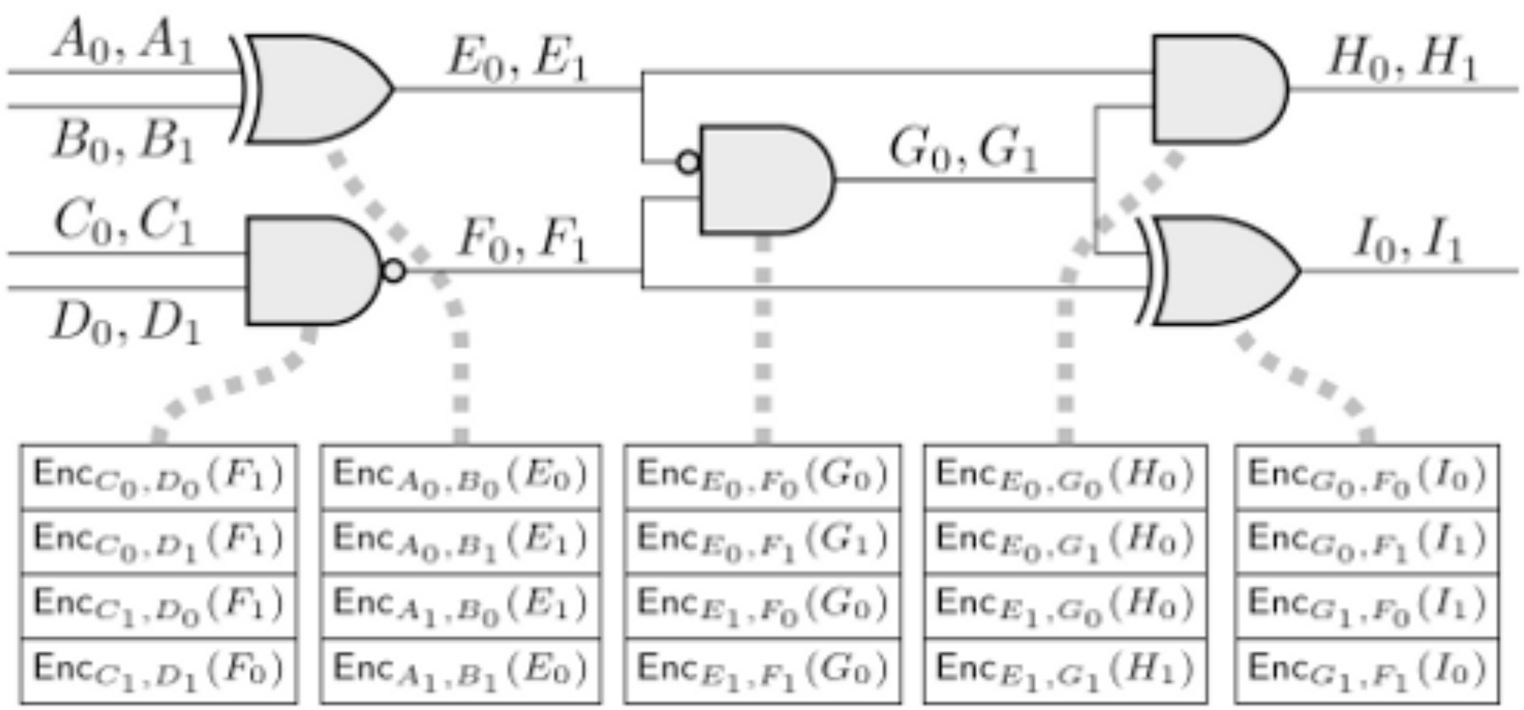

Yao Garbled Circuit - Secure 2-party Computing

Secret input a

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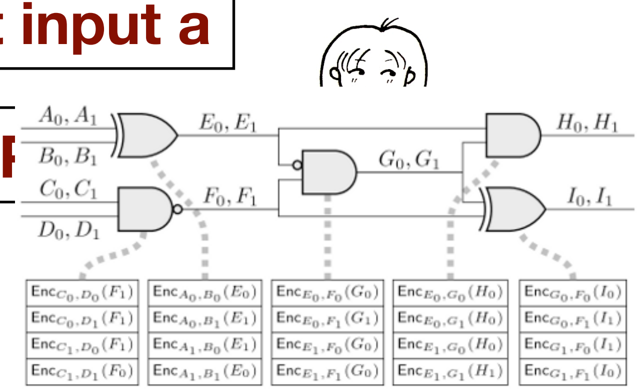

Yao Garbled Circuit - Secure 2-party Computing

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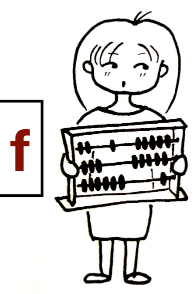
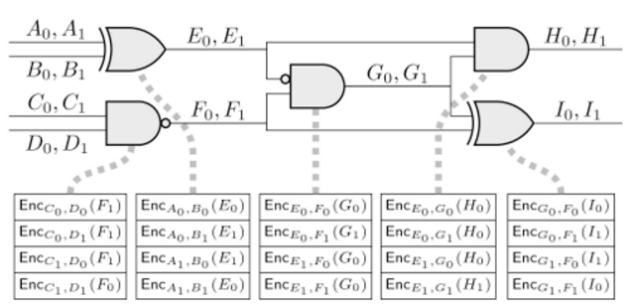
Garbled Program f





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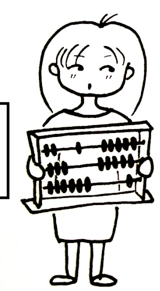
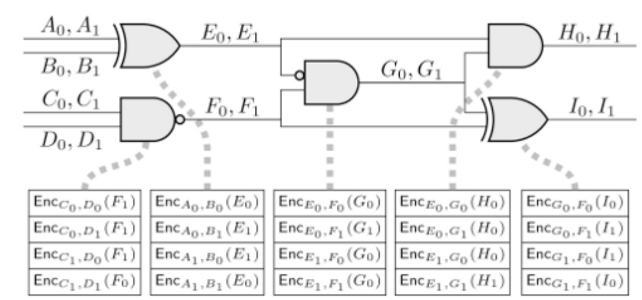


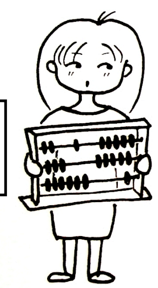


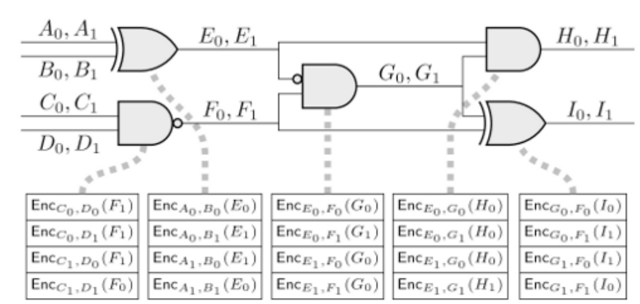
Insert secret input b
Evaluate f(a,b)

Yao Garbled Circuit - Secure 2-party Computing

Secret input a

Garbled Program f





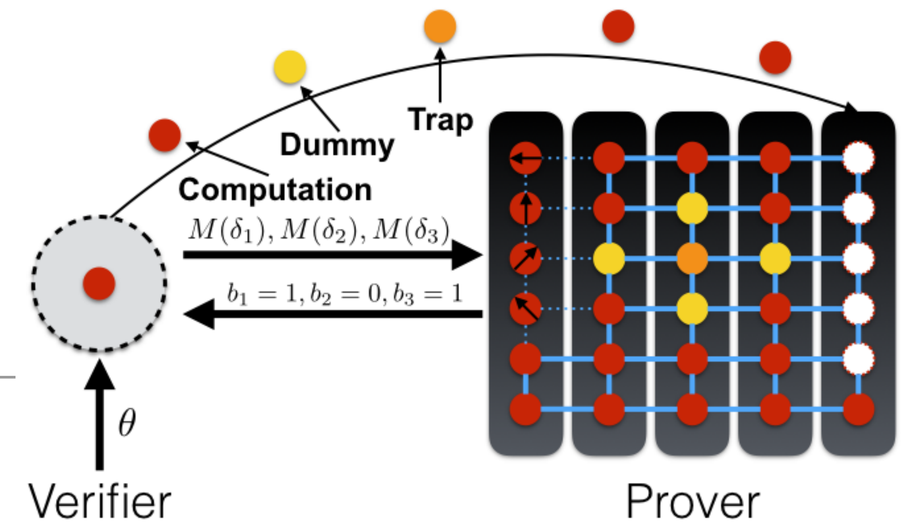
Insert secret input b
Evaluate f(a,b)

Computational Security

Requires OT

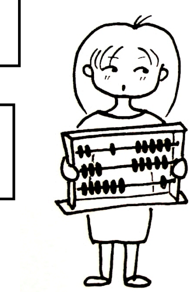
Honest but Curious Adversary

Verifiable Quantum Yao



Secret input q_c

Garbled CP map



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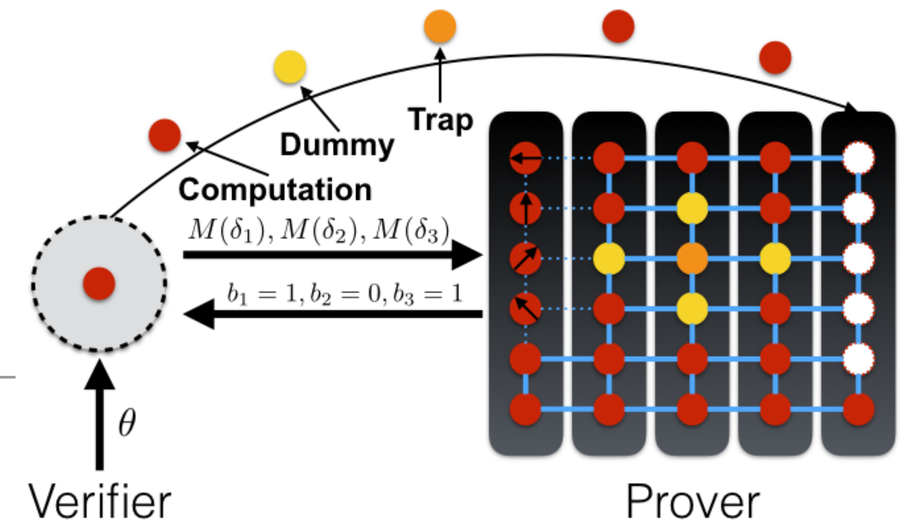

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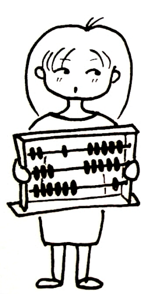
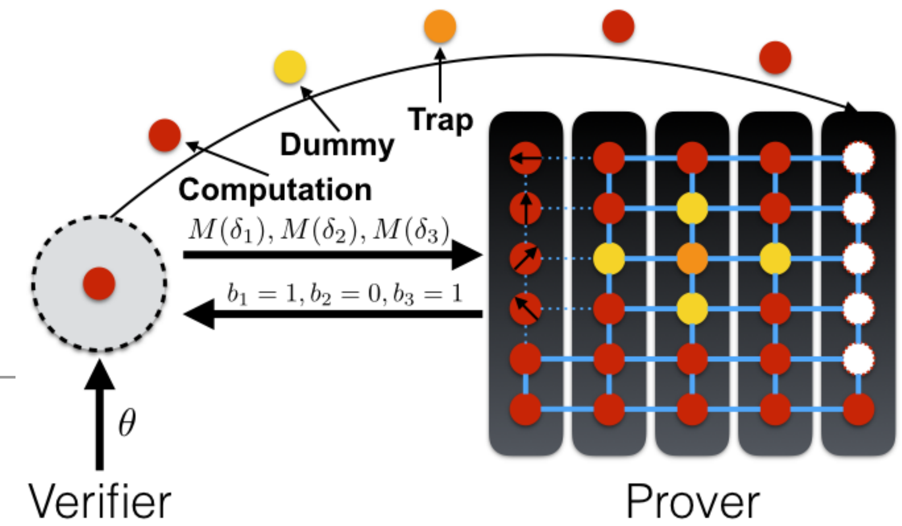


θ $\cdots$ θ'

$$1/\sqrt{2}\left(|0\rangle + e^{i\theta}|1\rangle\right)$$

$$|0\rangle, |1\rangle$$

Verifiable Quantum Yao



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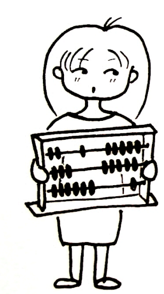

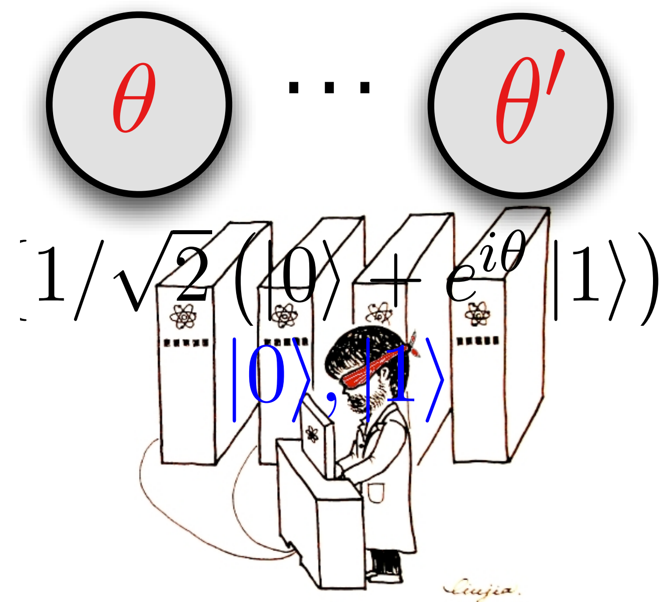

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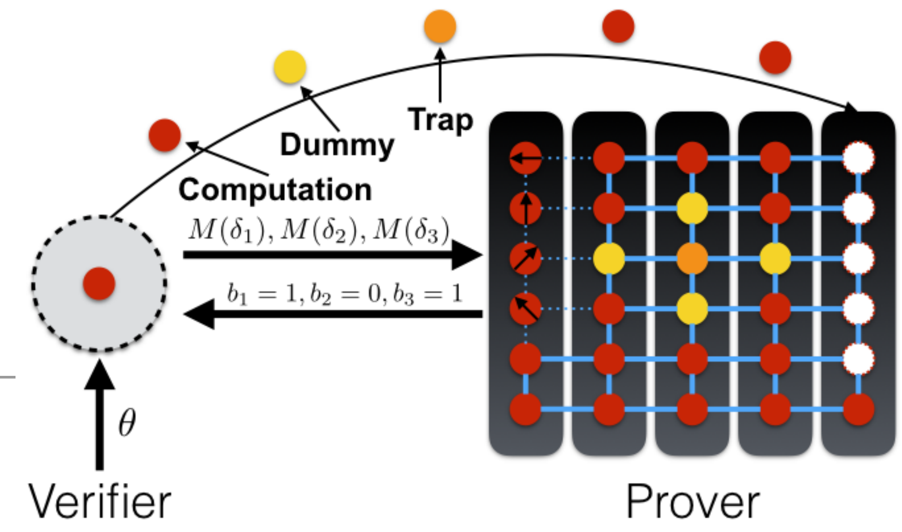

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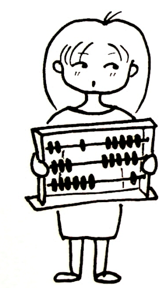

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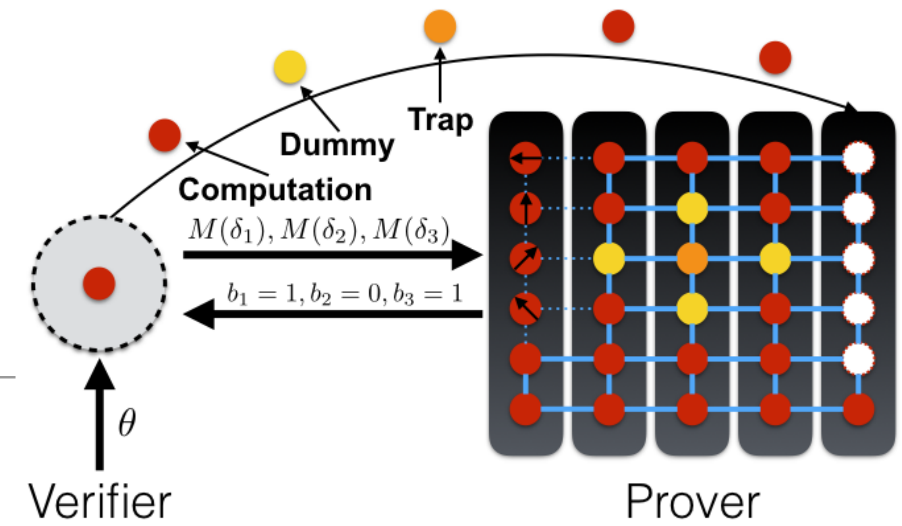

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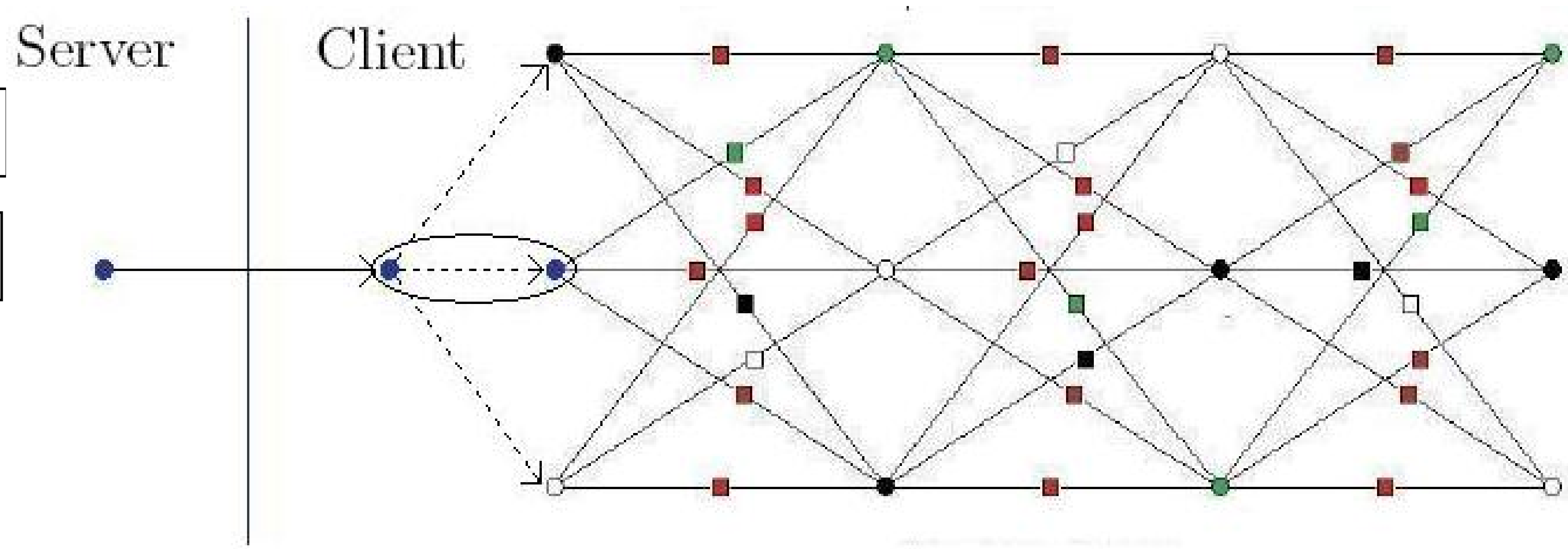
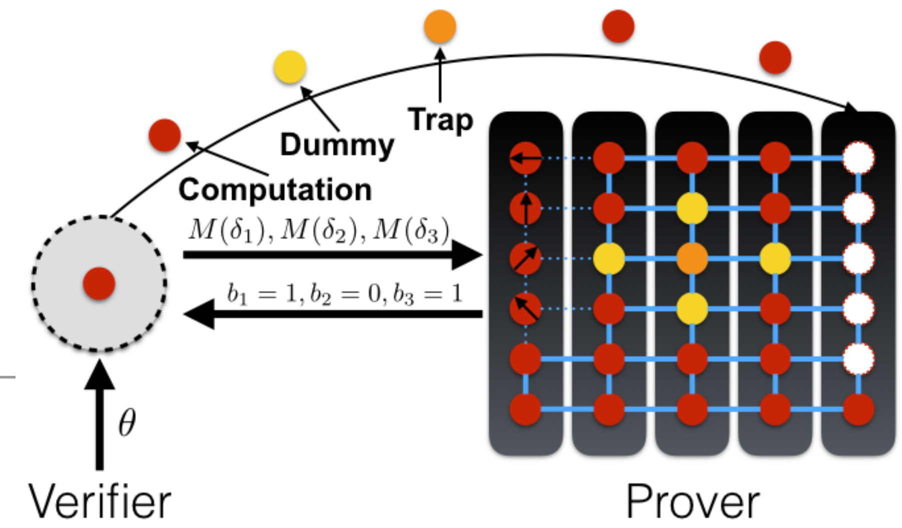


Server's input placed in DT(G) with correspoding trap-colouring

Verifiable Quantum Yao



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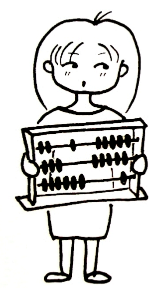

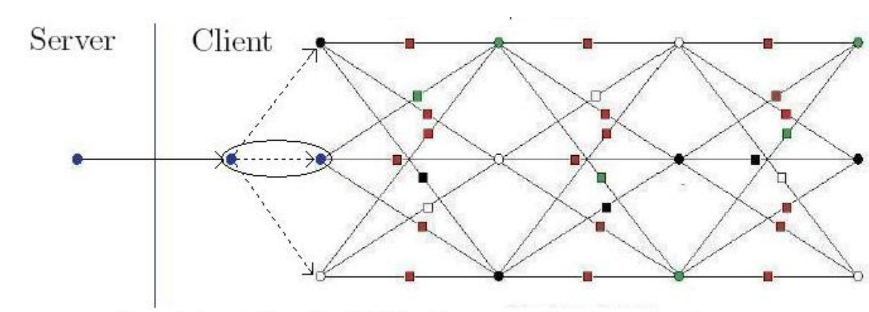

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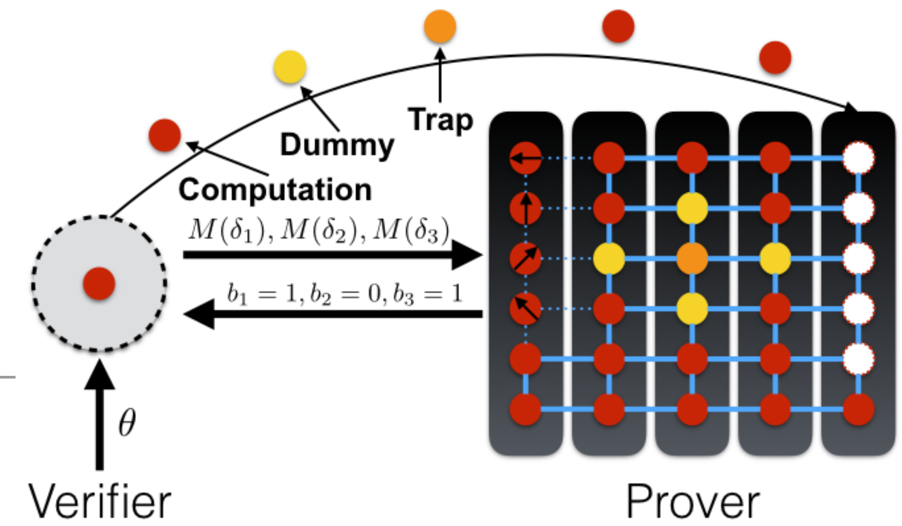

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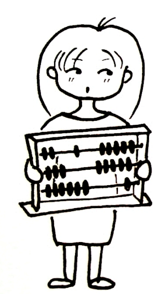
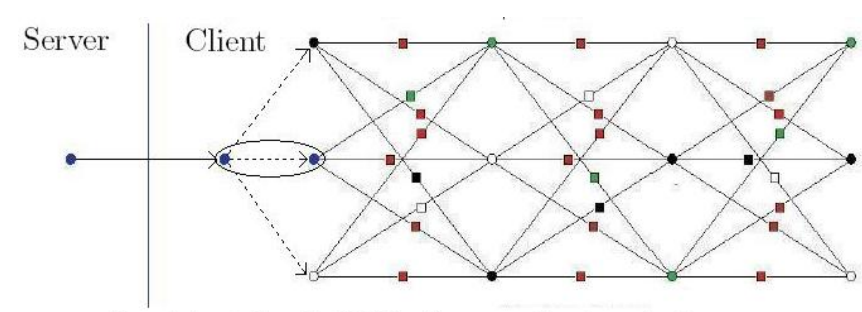


$r_{x,y} \in_R \{0,1\}$

$$\delta_{x,y} = \phi'_{x,y} + \theta_{x,y} + \pi r_{x,y}$$



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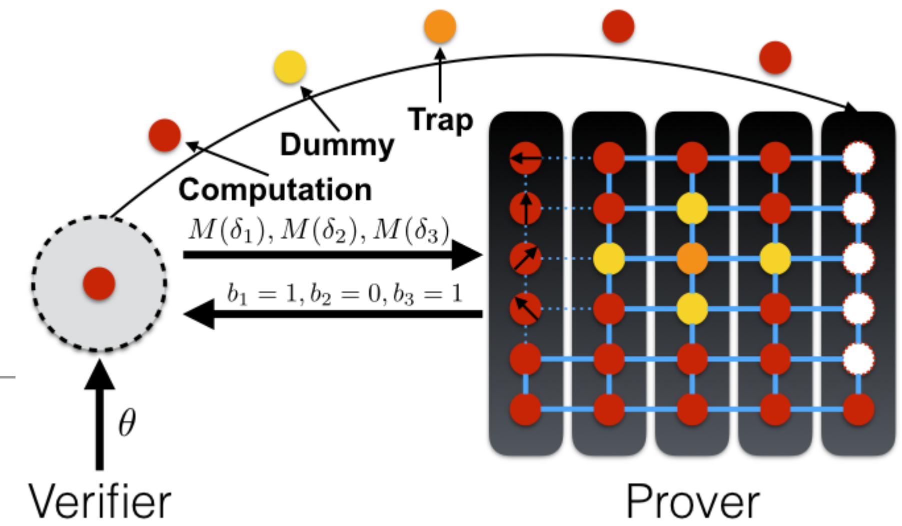

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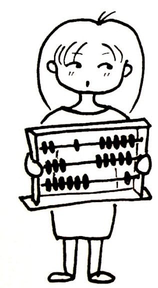

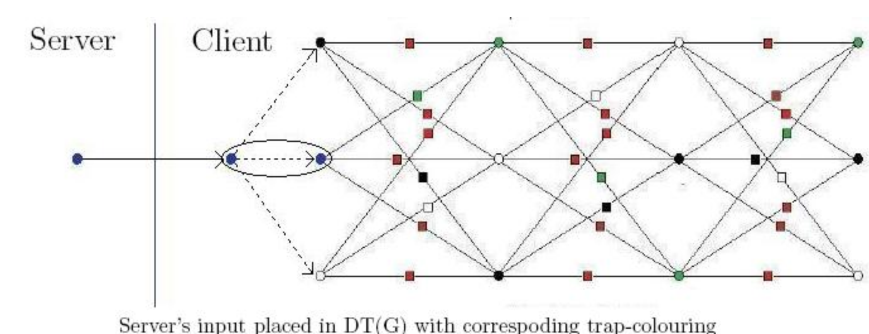

$r_{x,y} \in_R \{0,1\}$

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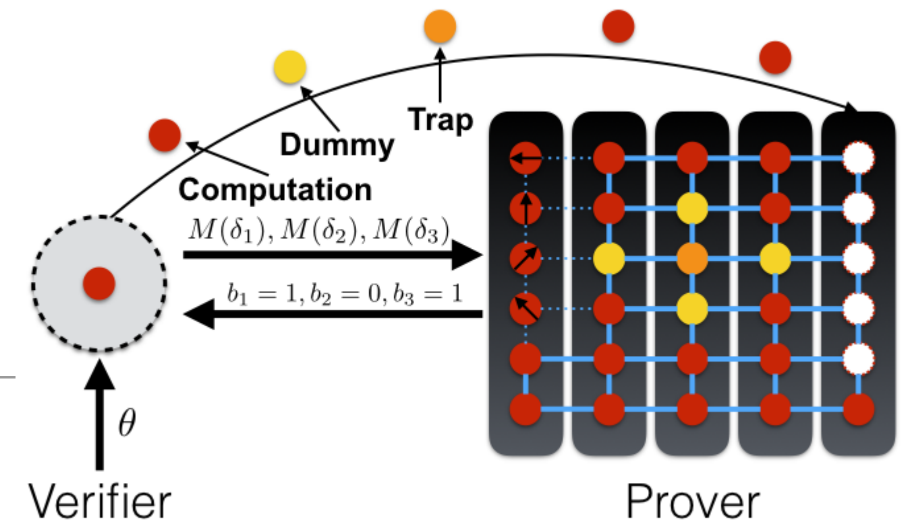
Insert secret input q_c
Evaluate CP(q_c,q_s)

Verifiable Quantum Yao



Secret input q_c

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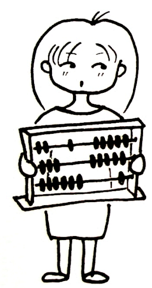

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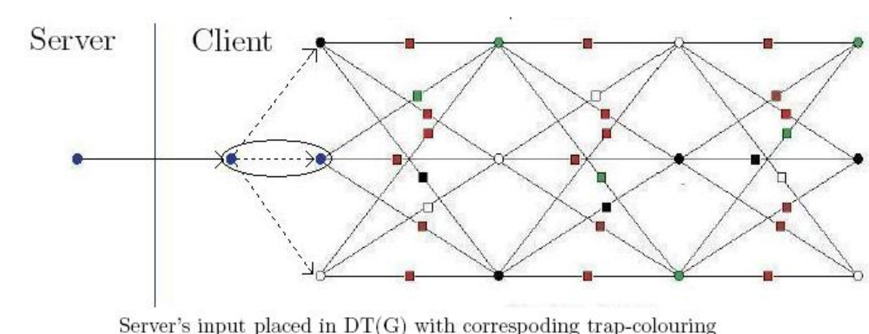


Insert secret input q_c
Evaluate CP(q_c,q_s)

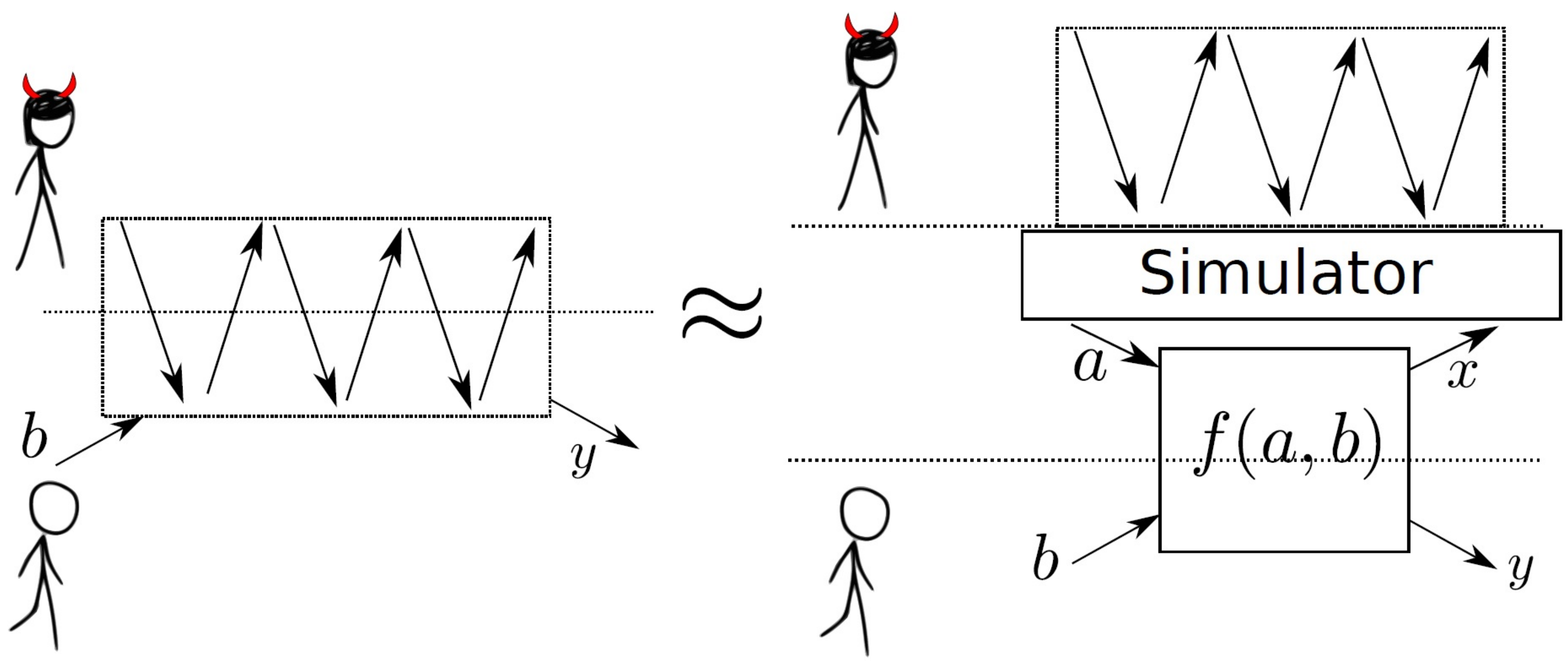
Unconditional Security

no OT is needed

Quantum Honest but Curious Client

Requires classical O(N) online communication

Security Model



The adversary cannot distinguish between the actual protocol

or

interacting with the ideal functionality and the simulator

Quantum Adversaries

Malicious Server: Can deviate in any possible quantum way

Specious Client: Can deviate in any way, provided that for every step of the protocol they can reproduce the honest state of that step by acting only on their system. i.e. can pass an audit at all steps of the protocol.

Quantum Adversaries

Malicious Server: Can deviate in any possible quantum way

Specious Client: Can deviate in any way, provided that for every step of the protocol they can reproduce the honest state of that step by acting only on their system. i.e. can pass an audit at all steps of the protocol.

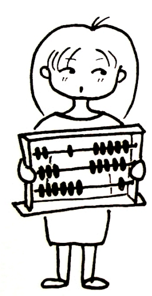
Formally, an adversary $\mathcal{A}$ is ϵ-specious if there exists a family of CP-maps $\mathcal{T}_i : L(\tilde{\mathcal{A}}_i) \to L(\mathcal{A}_i)$ one for each step i of the protocol such that for *every* allowed input ρ_{in}

$$\Delta(\mathcal{T}_i \otimes \mathbb{I} \cdot \tilde{\rho}_i(\tilde{A}, \rho_{in}), \rho_i(\rho_{in})) \leq \epsilon$$

where $\rho_i(\rho_{in})$ is the honest state at step i and $\tilde{\rho}_i(\tilde{A}, \rho_{in})$ the state of the real (deviated) protocol at the same step.

Malicious Client with Cut and Choose

1-way Q communication
Client is QKD
Linear Q + Poly C overhead

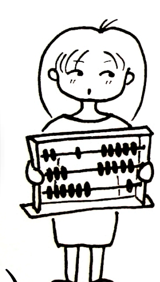


Malicious Client with Cut and Choose

com(r), com(θ),
com(δ),
com(δ_{input}),
com(keys for P_2),
com(position of
traps in final)

1-way Q communication
Client is QKD
Linear Q + Poly C overhead

θ $\cdots$ θ'



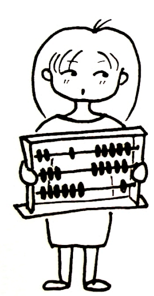
$1/\sqrt{2}\left(|0\rangle + e^{i\theta}|1\rangle\right)$

$|0\rangle, |1\rangle$

commitment to *S* version of circuit

Malicious Client with Cut and Choose

1-way Q communication
Client is QKD
Linear Q + Poly C overhead



com(r), com(θ), com(δ), com(δ_{input}), com(keys for P_2), com(position of traps in final)

θ $\cdots$ θ'

$$1/\sqrt{2}\left(|0\rangle + e^{i\theta}|1\rangle\right)$$

$$|0\rangle, |1\rangle$$

commitment to *S* version of

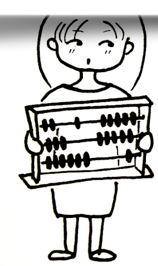
Kashefi Music Wallden 17

Malicious Client with Cut and Choose

1-way Q communication
Client is QKD
Linear Q + Poly C overhead

com(r), com(θ),
com(δ),
com(δ_{input}),
com(keys for P_2),
com(position of
traps in final)

coin tossing protocol to decide the evaluation graph



θ $\cdots$ θ'

$$1/\sqrt{2}\left(|0\rangle + e^{i\theta}|1\rangle\right)$$

$$|0\rangle, |1\rangle$$

commitment to *S* version of

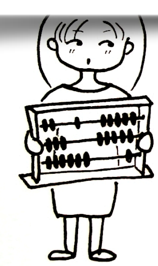
Kashefi Music Wallden 17

Malicious Client with Cut and Choose

1-way Q communication
Client is QKD
Linear Q + Poly C overhead

$\mathrm{com(r)}, \mathrm{com}(\theta),$
$\mathrm{com}(\delta),$
$\mathrm{com}(\delta_{input}),$
$\mathrm{com(keys\ for\ P_2)},$
$\mathrm{com(position\ of}$
$\mathrm{traps\ in\ final)}$

coin tossing protocol to decide the evaluation graph



$\theta \quad \cdots \quad \theta'$

$$1/\sqrt{2}\left(|0\rangle + e^{i\theta}\,|1\rangle\right)$$

$|0\rangle, |1\rangle$

Server checks Client

commitment to *S* version of

Kashefi Music Wallden 17

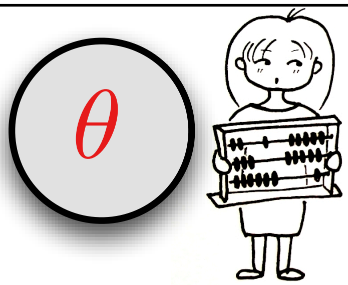
Malicious Client and Server with Cut and Choose

- Client chooses values for circuits
- Client creates commitments
- OT protocols => Server gets his inputs
- Client prepares and sends qubits
- Client sends commitments
- Coin-tossing protocol => Eval graph chosen
- Client decommits for the check graphs
- Server performs consistency checks
- Server run VUBQC protocol
- Key exchange protocol

Secure Multi Party Quantum Computing

Secret input q_1

Garbled her part of the CP map



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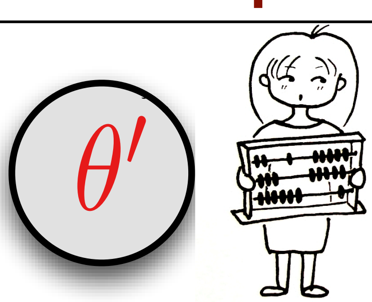
Secret input q_n

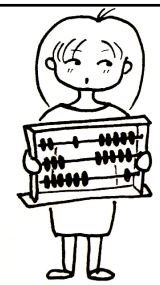
Garbled her part of the CP map



Secure Multi Party Quantum Computing

Secret input q_1

Garbled her part of the CP map



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Secret input q_n

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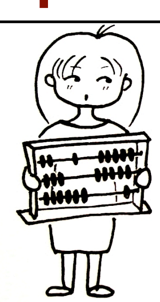


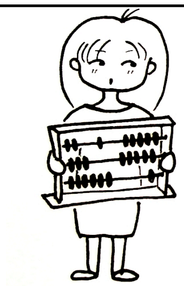
θ

θ'

Secure Multi Party Quantum Computing

Secret input q_1

Garbled her part of the CP map



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Secret input q_n

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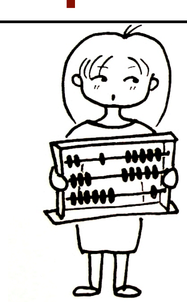

Secure Multi Party Quantum Computing

Secret input q_1

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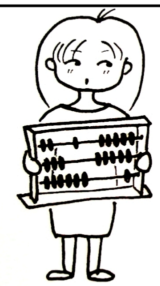

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Secret input q_n

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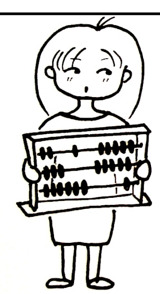
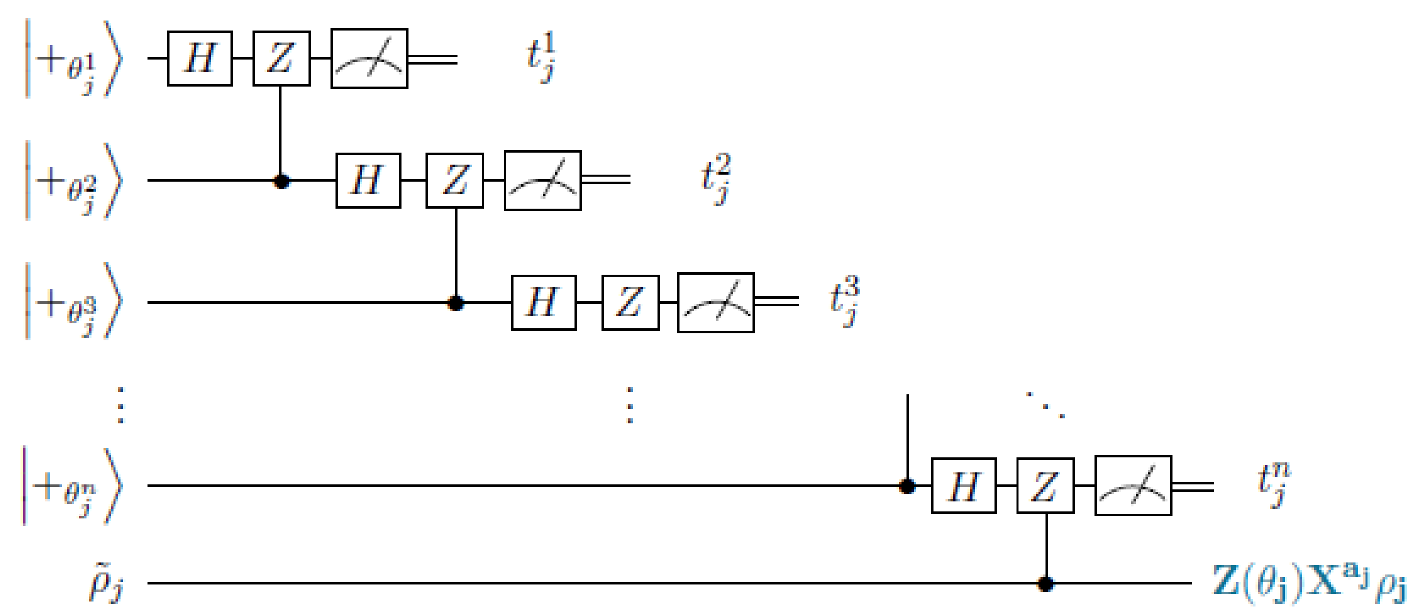


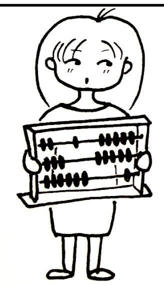


$$\theta_j = \theta_j^j + \sum_{k=1,k\neq j}^{n} (-1)^{\bigoplus_{i=k}^{n} t_j^i}\, \theta_j^k$$

Secure Multi Party Quantum Computing

Secret input q_1

Garbled her part of the CP map



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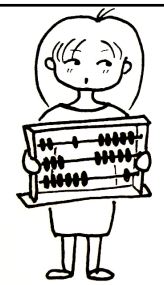
Secret input q_n

Garbled her part of the CP map



Secure Multi Party Quantum Computing

Secret input q_1

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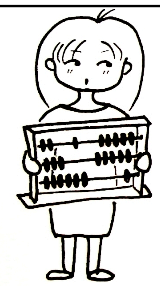

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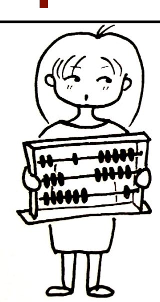
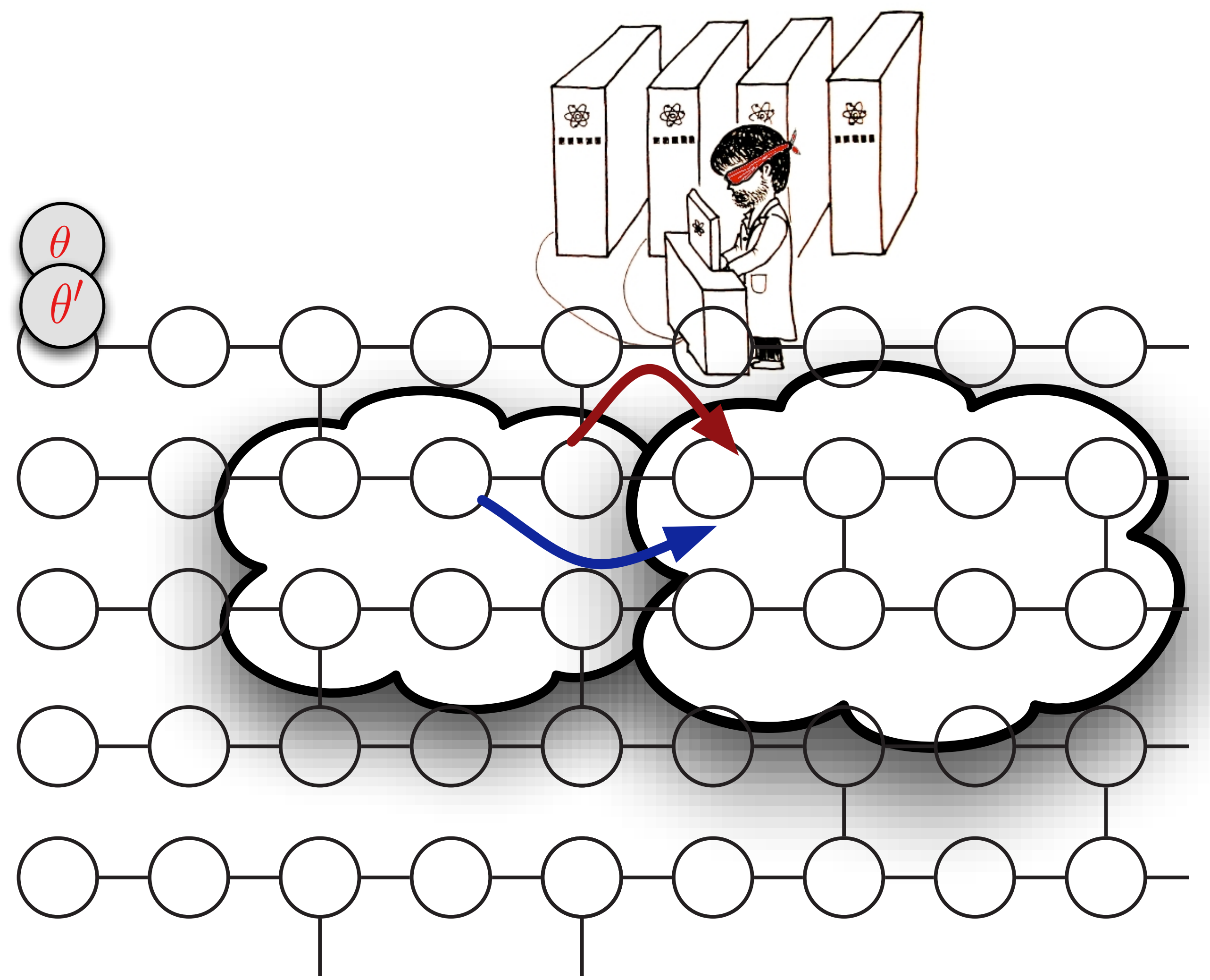
Secret input q_n

Garbled her part of the CP map





Secure Multi Party Quantum Computing

Secret input q_1

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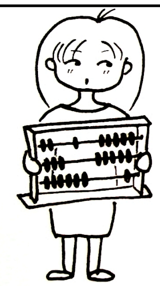

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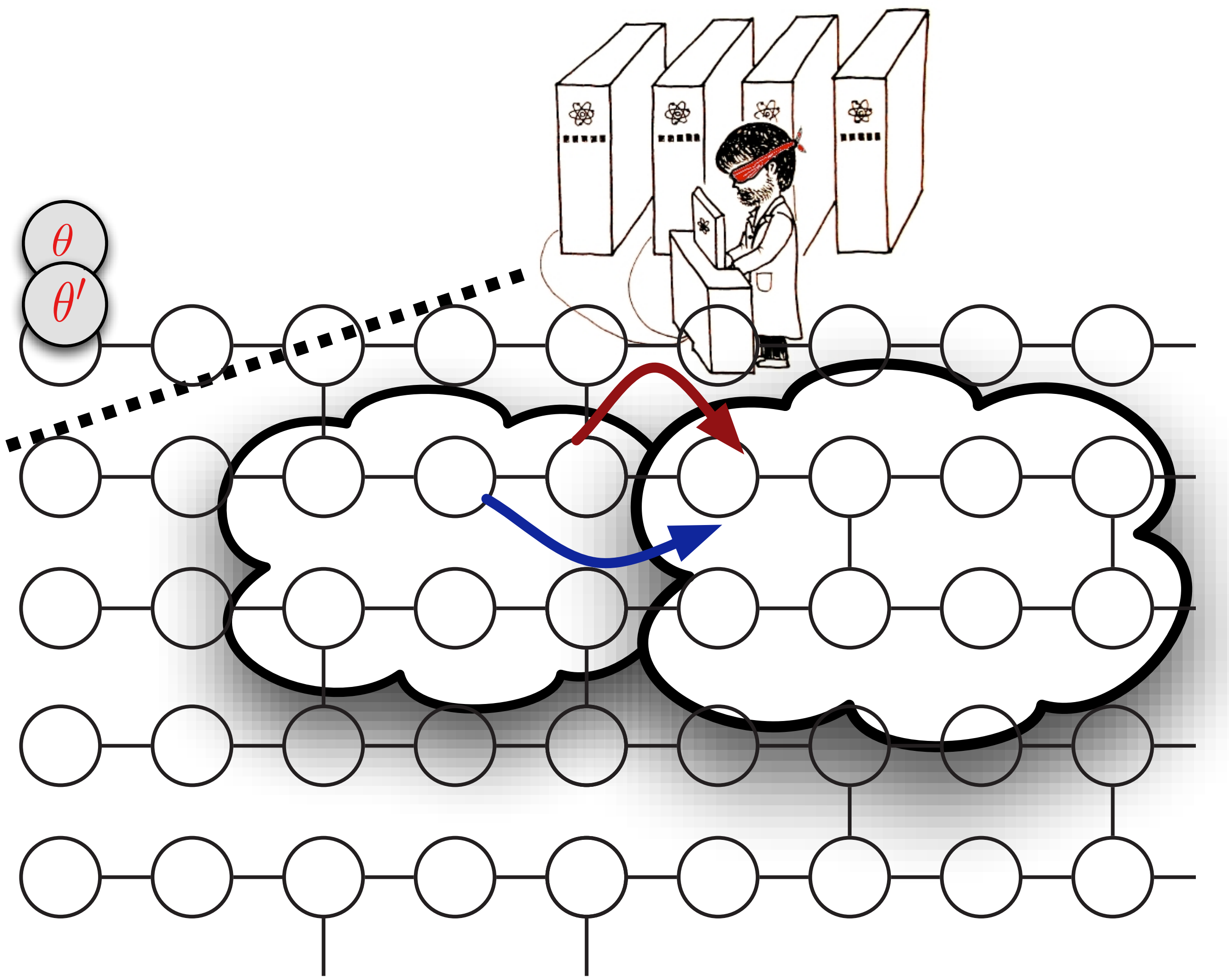
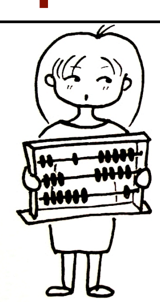
$$\delta_j = \phi'_j + \pi \bigoplus_{k=1}^{n} r_j^k + \theta_j$$

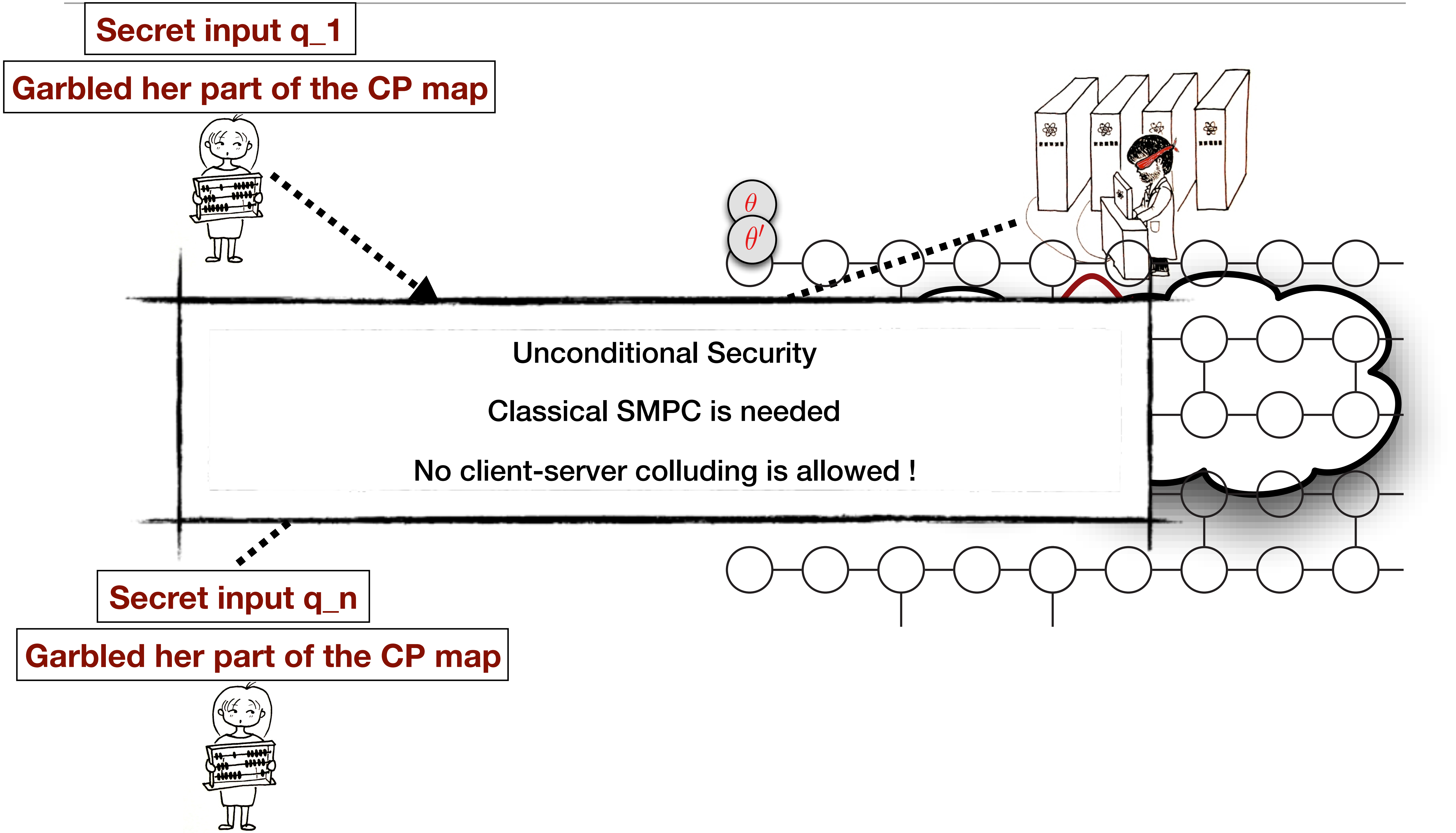


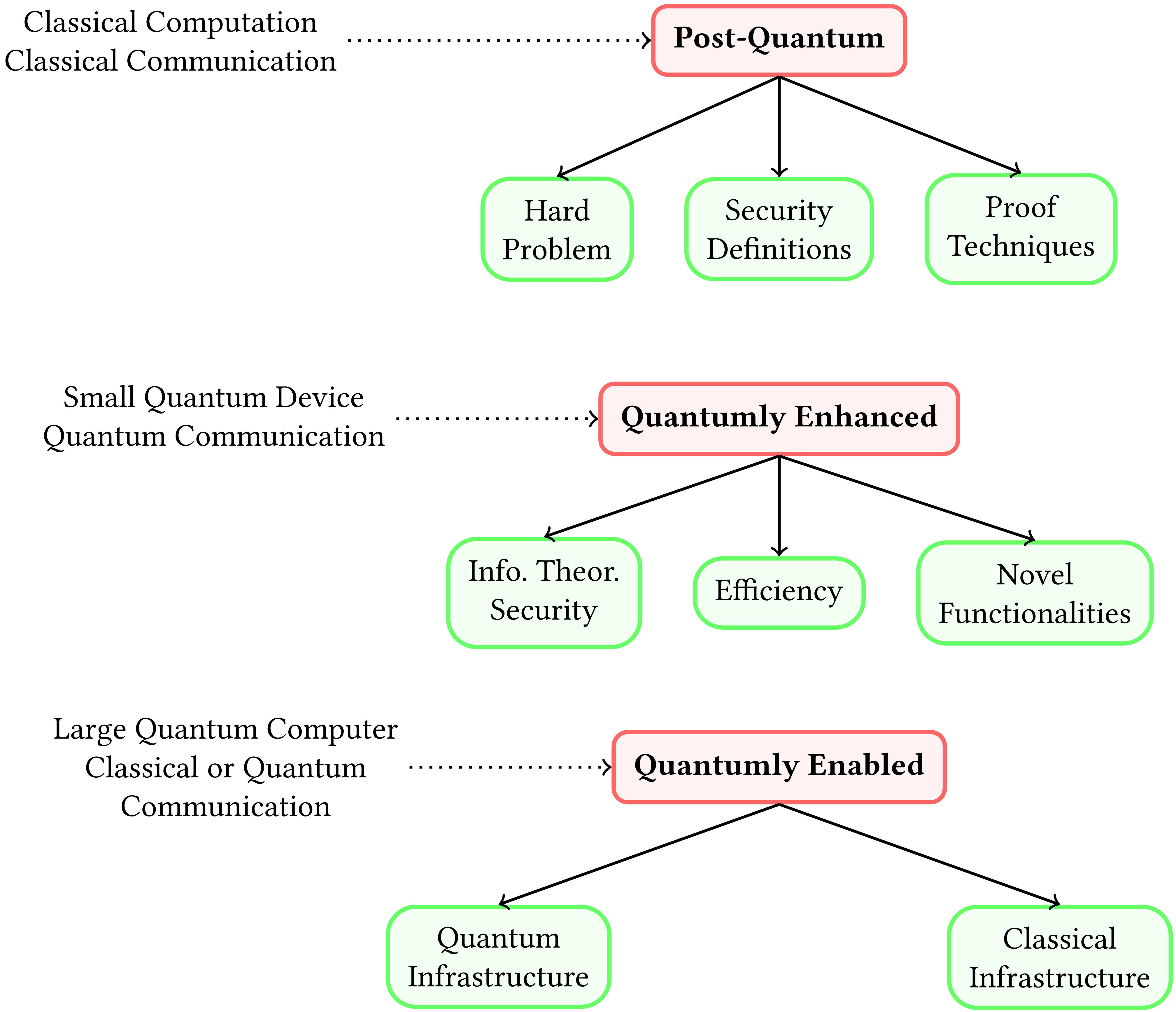
Secret input q_n

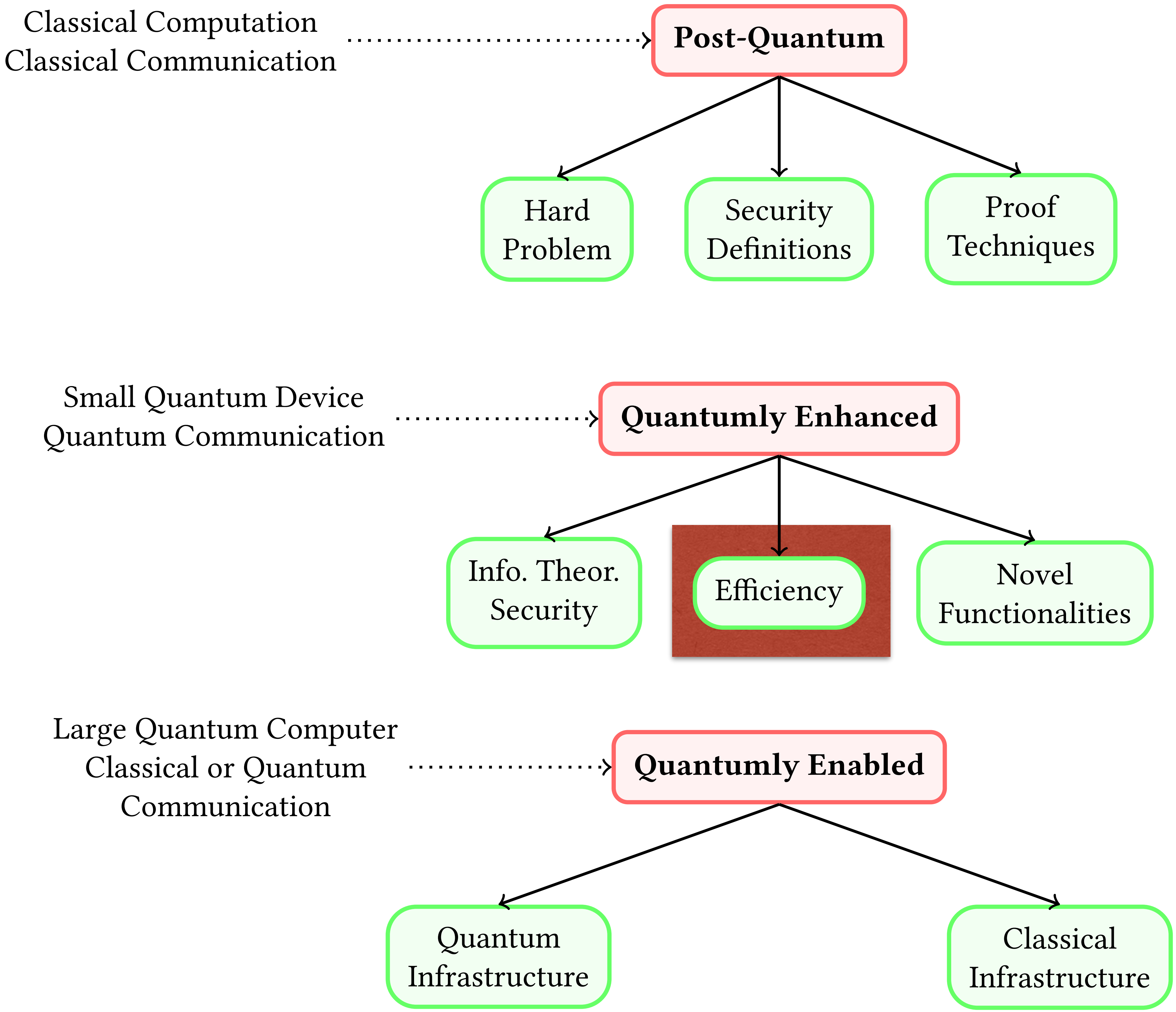
Garbled her part of the CP map



Secure Multi Party Quantum Computing




Classical Computation
Classical Communication
Post-Quantum
Hard Problem
Security Definitions
Proof Techniques
Small Quantum Device
Quantum Communication
Quantumly Enhanced
Info. Theor. Security
Efficiency
Novel Functionalities
Large Quantum Computer
Classical or Quantum Communication
Quantumly Enabled
Quantum Infrastructure
Classical Infrastructure


Classical Computation
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Hard Problem
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Small Quantum Device
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Quantumly Enhanced
Info. Theor. Security
Efficiency
Novel Functionalities
Large Quantum Computer
Classical or Quantum Communication
Quantumly Enabled
Quantum Infrastructure
Classical Infrastructure

Practical Classical SMPC

First large-scale practical experiment with MPC to implement a secure auction

Bogetoftx- Christensen-Damgardz-Geislerz-Jakobsen-Krigaard-Nielsen-Nielseny-Pagter-Schwartzbachz-Toftyy08

Recently: Efficient (low communication) computational SMPC

Computation represented by a series of additions and multiplications of elements in F_p.

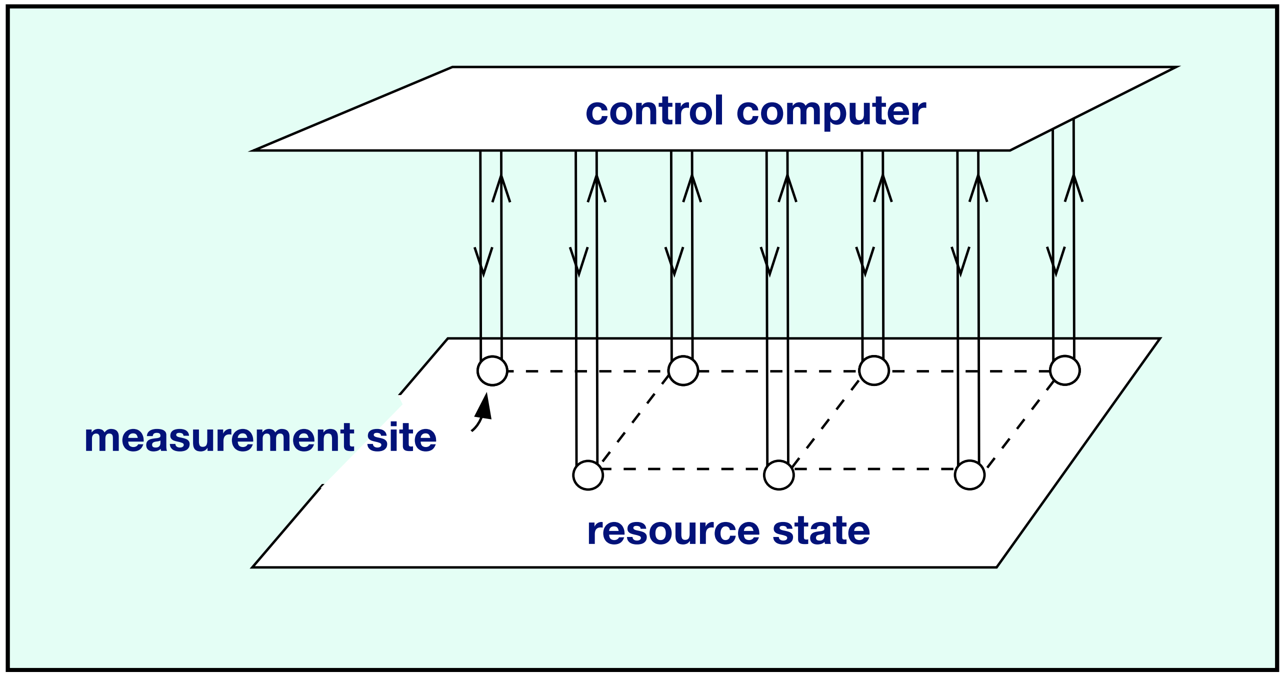
easy

Linear Verifiable Secret Sharing

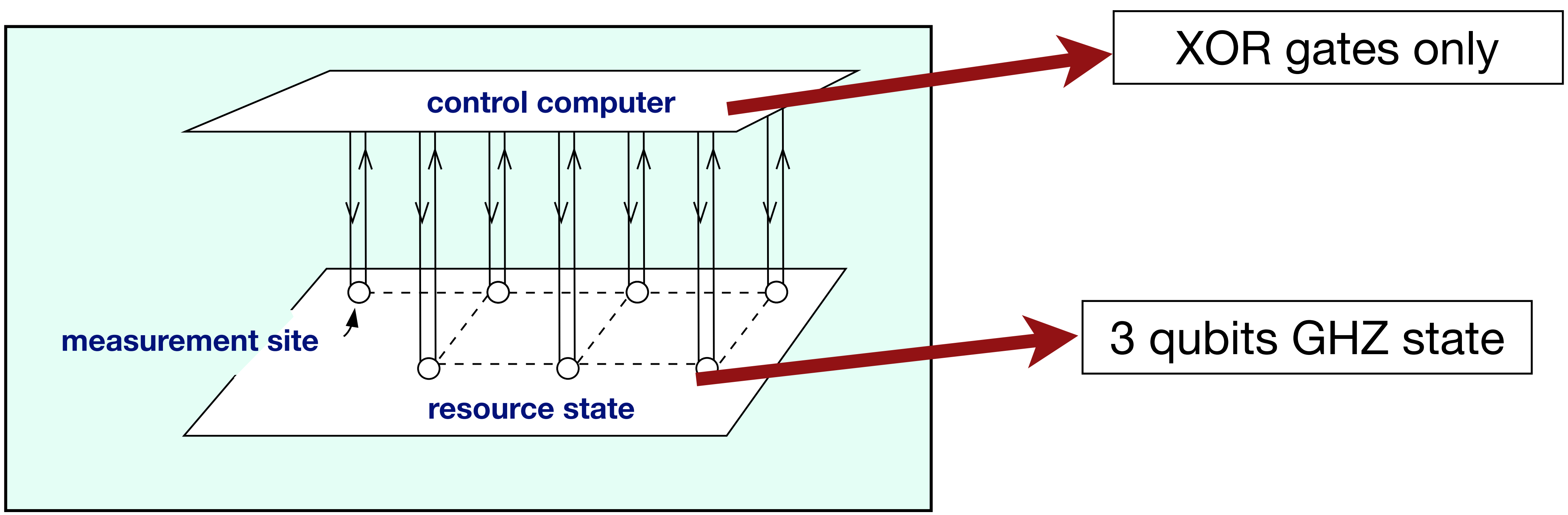
hard

costly but offline FHE

From Linear to Non-linear - MBQC



From Linear to Non-linear - MBQC



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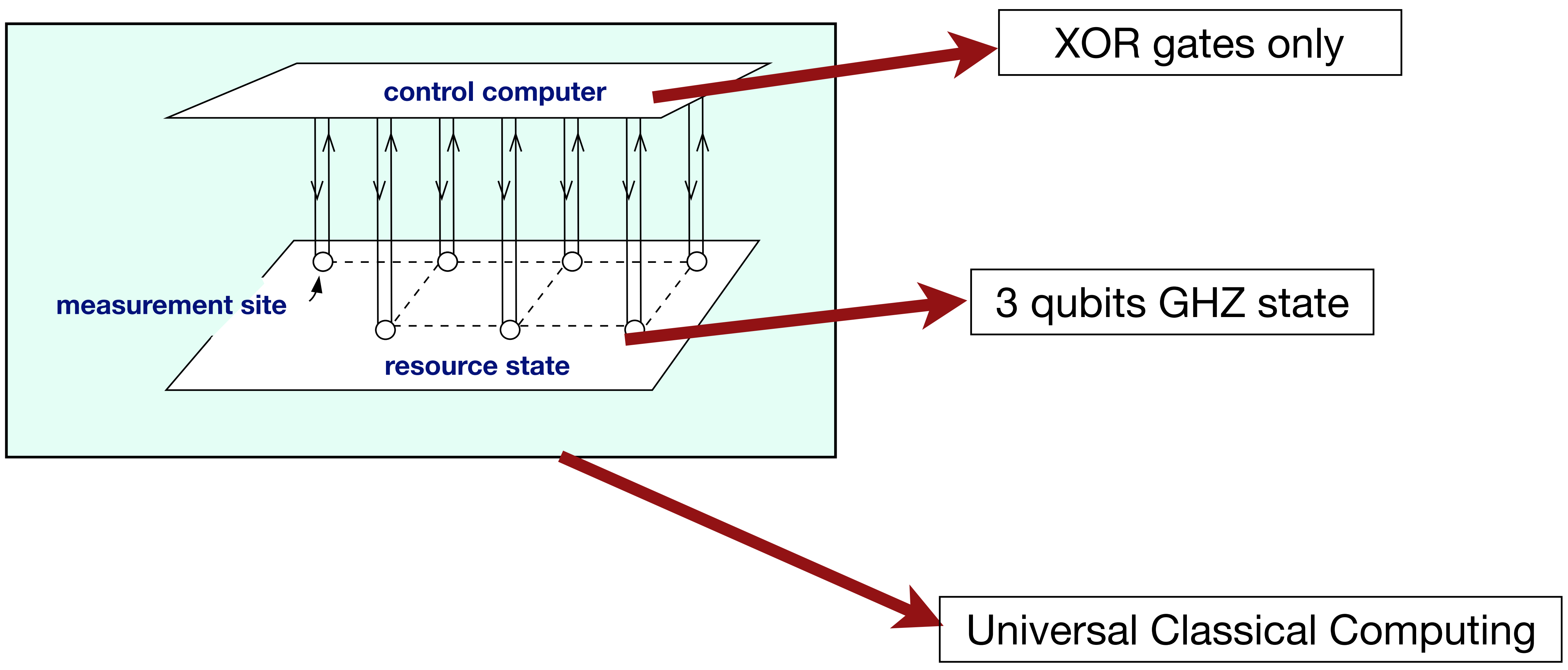

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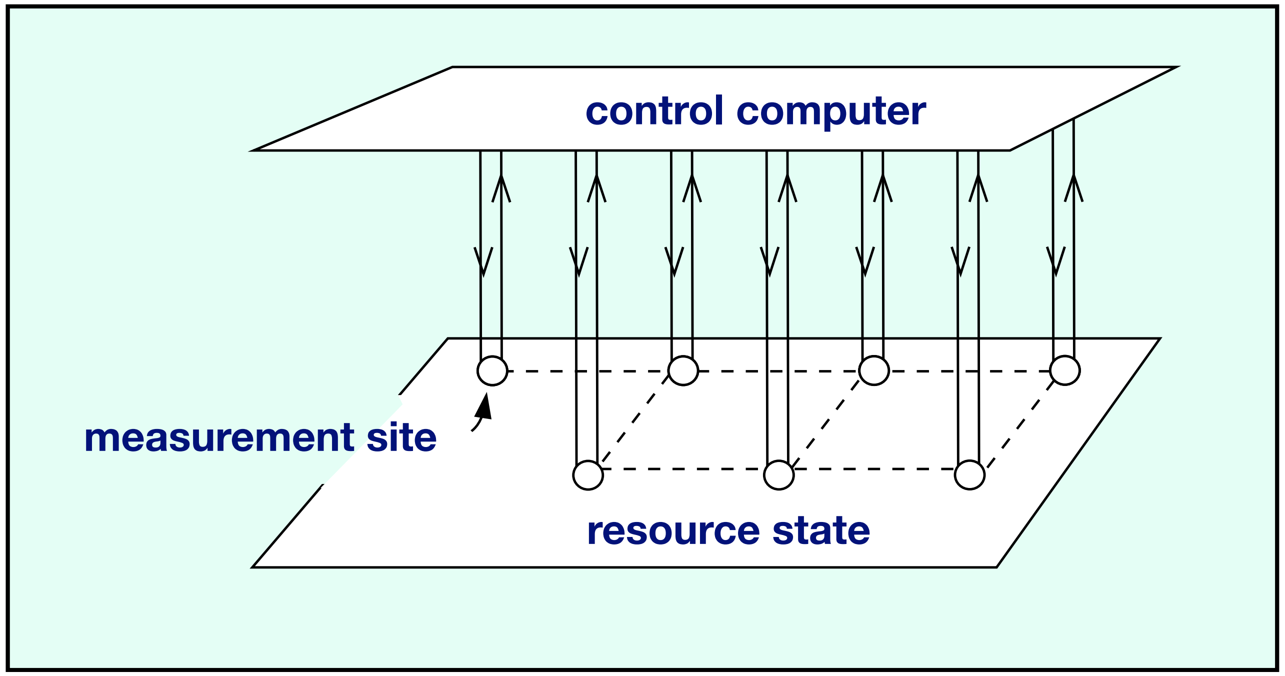

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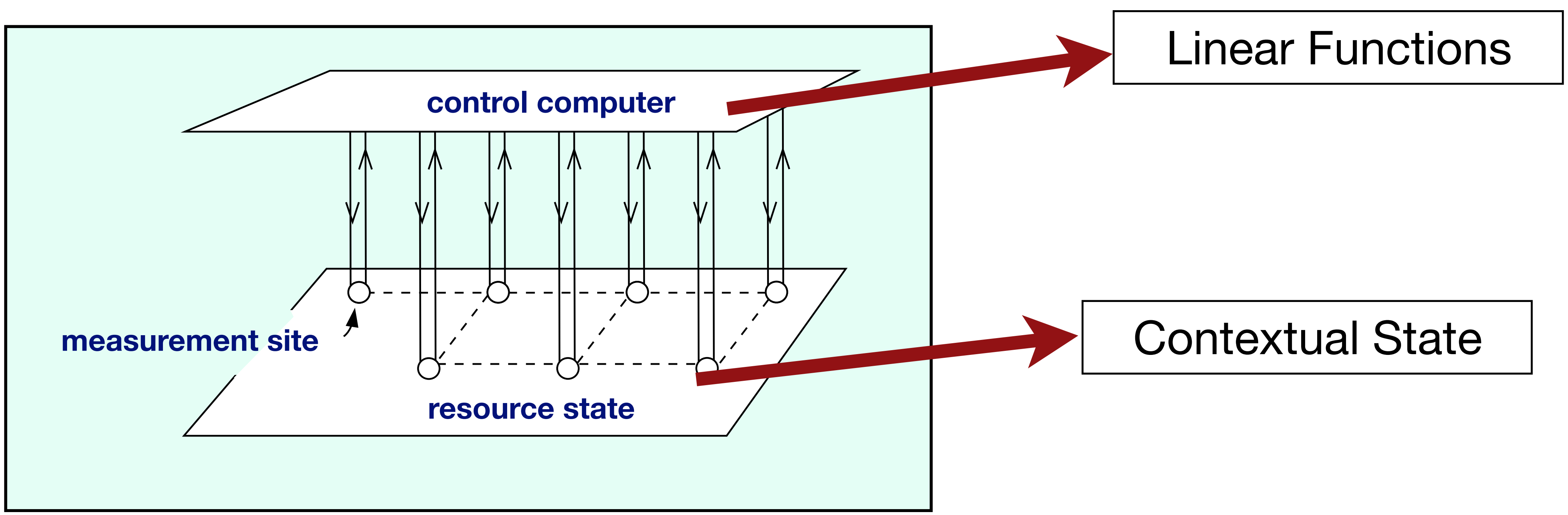

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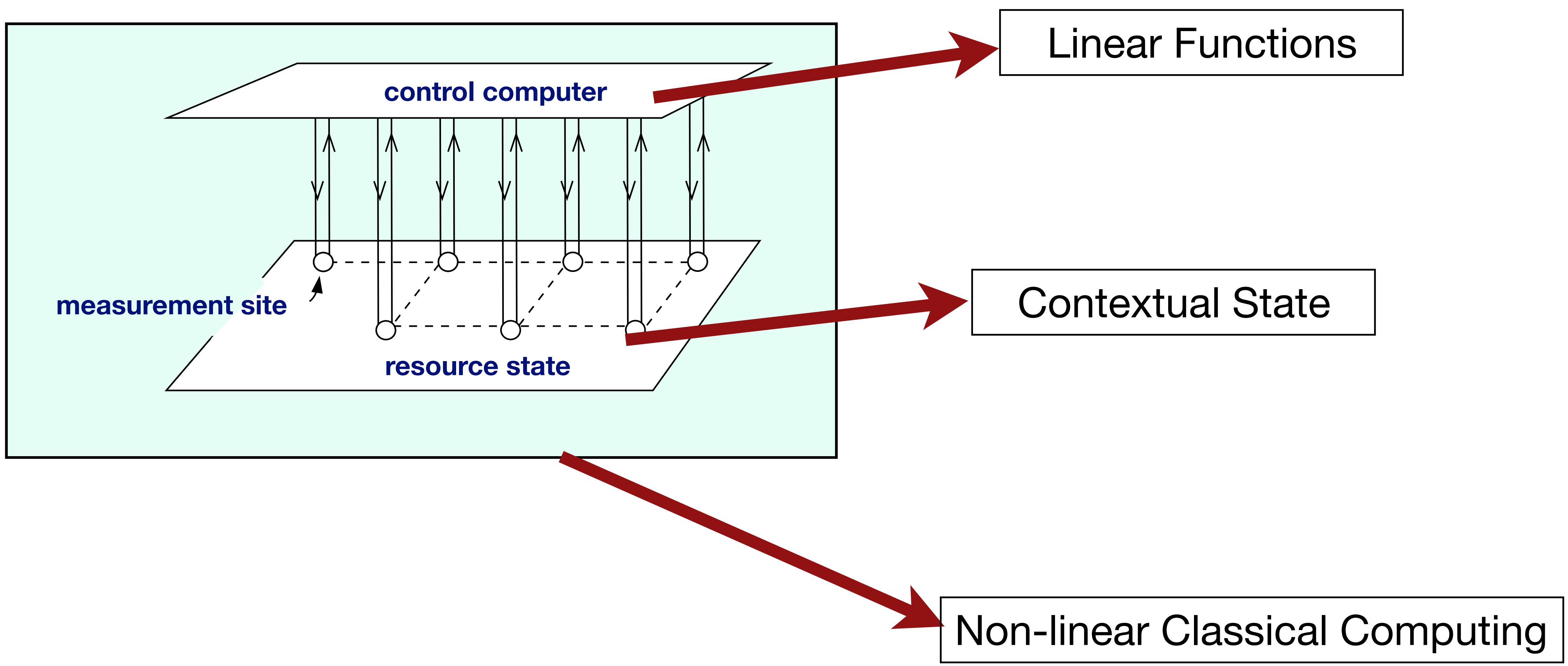
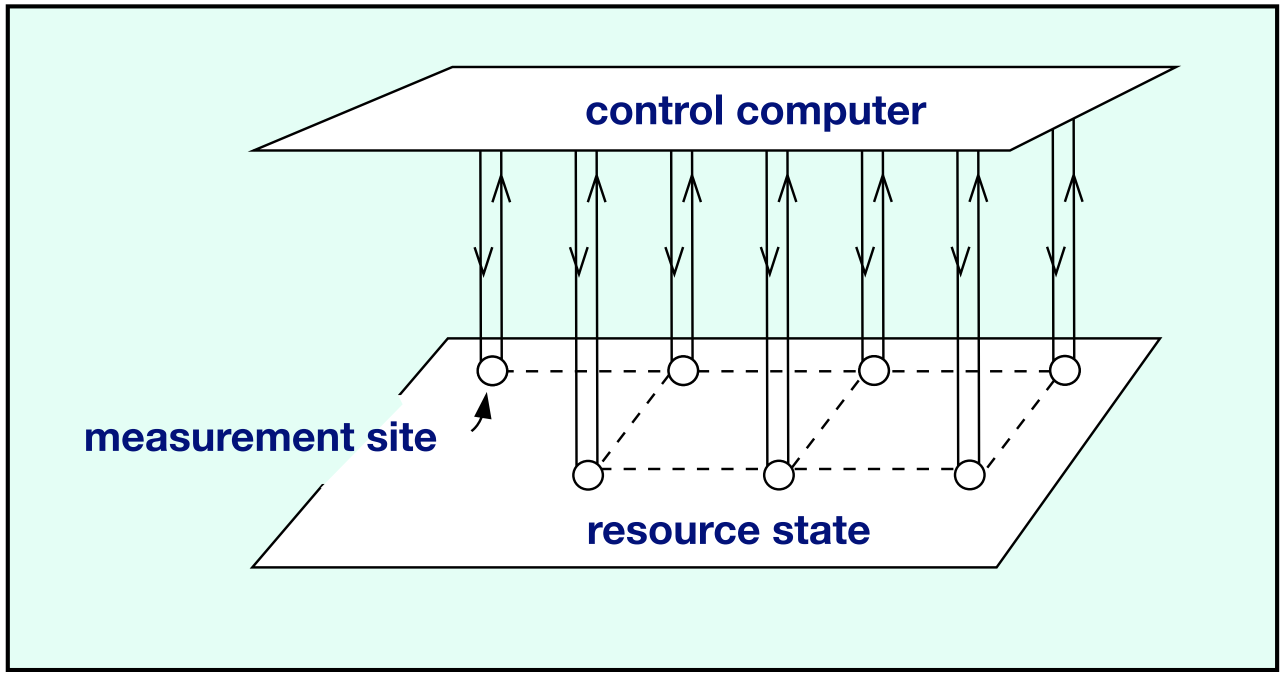


From Linear to Non-linear - Secure Computing



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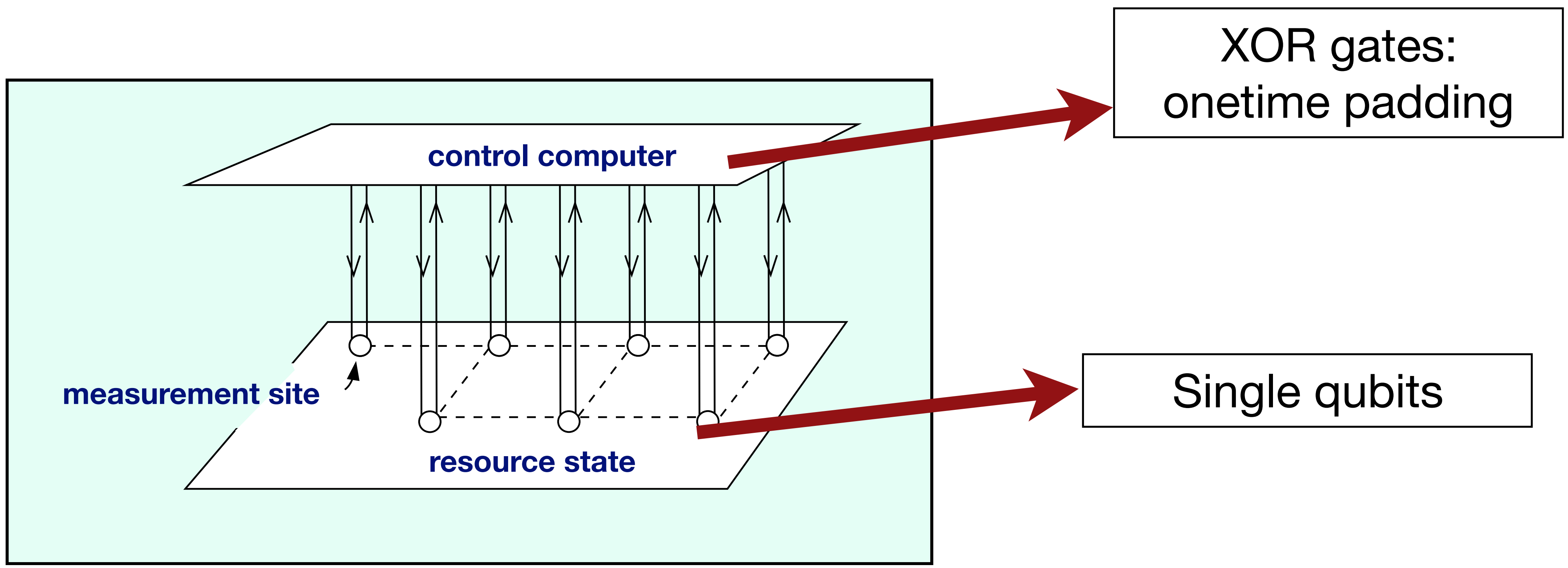

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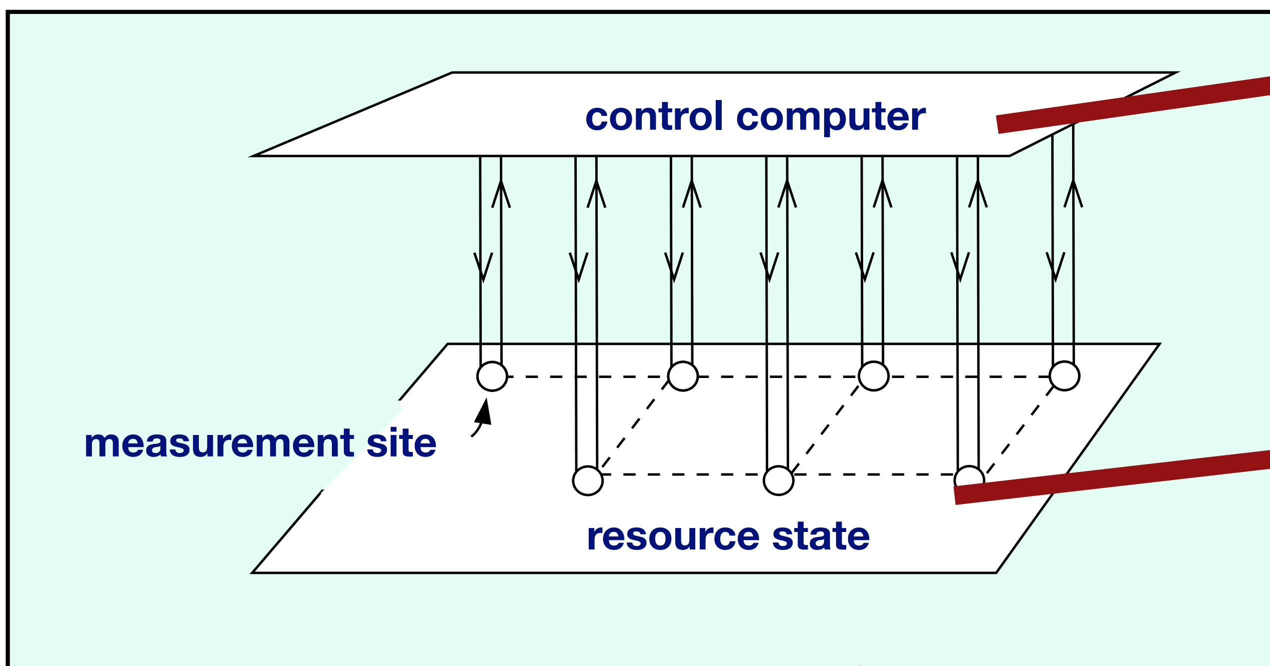

XOR gates:
onetime padding

Single qubits

Secure NAND

Restricted XOR Client

No classical protocol, with XOR client can securely delegate deterministic computation of NAND to a server.

Dunjko, Kapourniotis, Kashefi, arXiv:1405.4558, 2014

Restricted XOR Client

No classical protocol, with XOR client can securely delegate deterministic computation of NAND to a server.

Client's encoding: $C_1(a, b, \overrightarrow{x})$

Dunjko, Kapourniotis, Kashefi, arXiv:1405.4558, 2014

Restricted XOR Client

No classical protocol, with XOR client can securely delegate deterministic computation of NAND to a server.

Client's encoding: $C_1(\underbrace{a, b}_{\text{input}}, \overset{\text{random}}{\overrightarrow{x}})$ **XOR computable function independent of the input**

Dunjko, Kapourniotis, Kashefi, arXiv:1405.4558, 2014

Restricted XOR Client

No classical protocol, with XOR client can securely delegate deterministic computation of NAND to a server.

Client's encoding: $C_1(\underbrace{a, b}_{\text{input}}, \overbrace{\overrightarrow{x}}^{\text{random}})$ **XOR computable function independent of the input**

Server's computation: $S(C_1(a, b, \overrightarrow{x}))$

Dunjko, Kapourniotis, Kashefi, arXiv:1405.4558, 2014

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Server's computation: $S(C_1(a, b, \overrightarrow{x}))$

Client's decoding: $C_2(a, b, \overrightarrow{x}, S(C_1(a, b, \overrightarrow{x}))) = NAND(a, b)$ **XOR computable function**

Dunjko, Kapourniotis, Kashefi, arXiv:1405.4558, 2014

Restricted XOR Client

No classical protocol, with XOR client can securely delegate deterministic computation of NAND to a server.

Client's encoding: $\underbrace{C_1(a, b}_{\text{input}}, \overbrace{\overrightarrow{x}}^{\text{random}})$ **XOR computable function independent of the input**

Server's computation: $S(C_1(a, b, \overrightarrow{x}))$

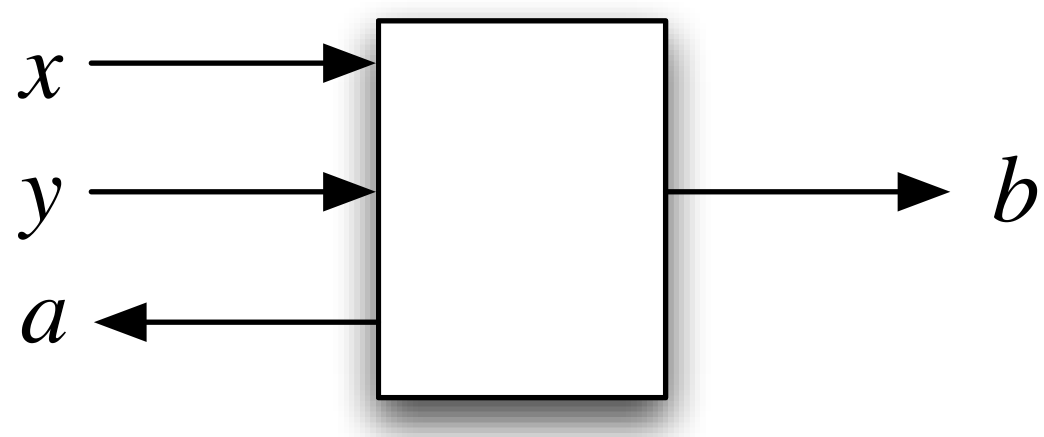
Client's decoding: $C_2(a, b, \overrightarrow{x}, \underbrace{S(C_1(a, b, \overrightarrow{x}))}_{\text{Constant}}) = NAND(a, b)$ **XOR computable function**

Dunjko, Kapourniotis, Kashefi, arXiv:1405.4558, 2014

Restricted XOR Client

No **quantum offline** protocol can delegate deterministically computation of NAND to a server while keeping the blindness

$$b = x.y + a$$



Quantum Communication

$$Z^r S^a S^b \left(S^\dagger\right)^{a \oplus b} |+\rangle \quad = \quad Z^r Z^{a \wedge b} |+\rangle$$

Quantum Communication

$$Z^r S^a S^b \left(S^\dagger\right)^{a\oplus b}|+\rangle = Z^r Z^{a\wedge b}|+\rangle$$

$$\begin{pmatrix}1 & 0\\ 0 & e^{ib\pi/2}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{ia\pi/2}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{-i(a\oplus b)\pi/2}\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & e^{-i(a\wedge b)\pi}\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix}$$

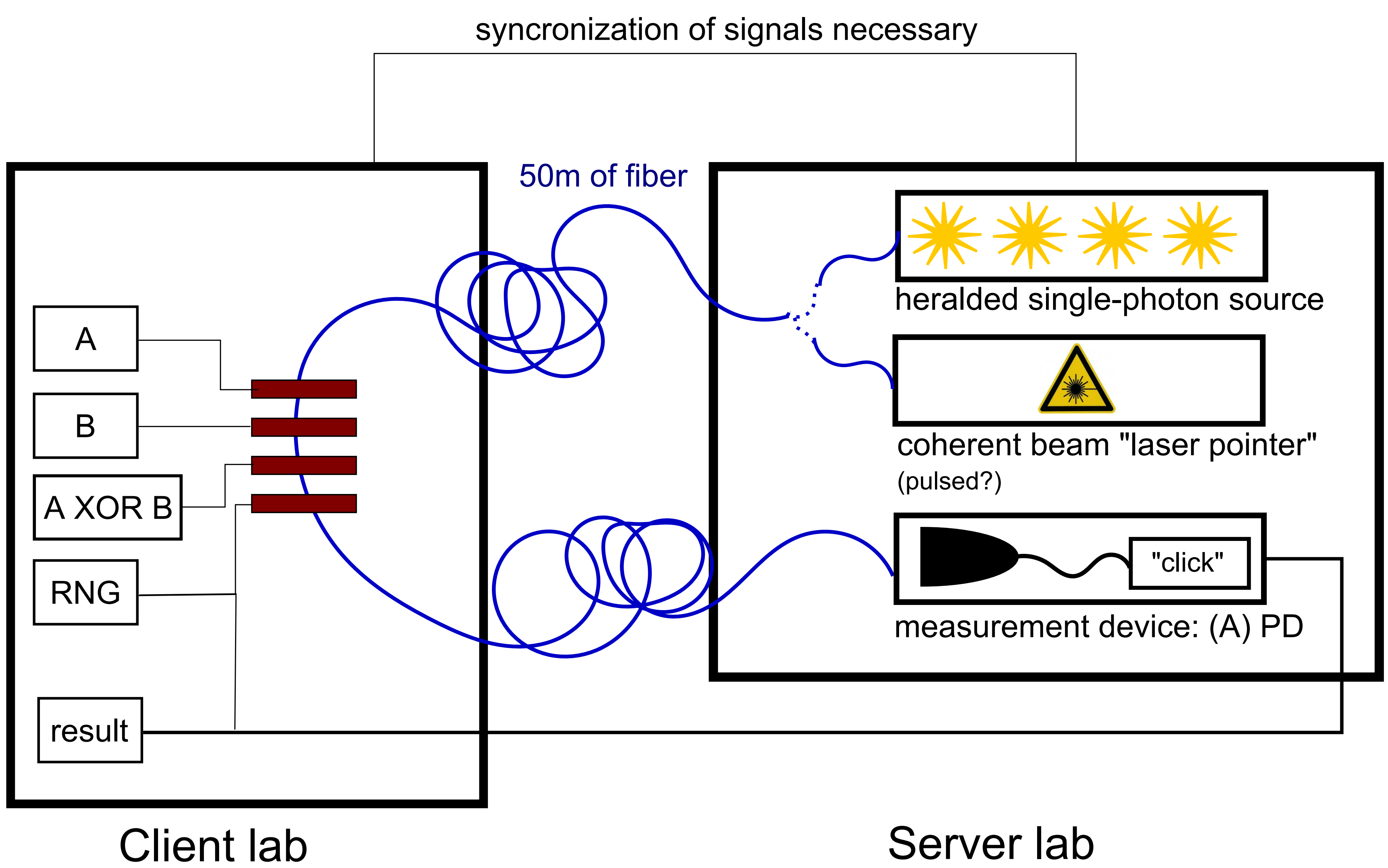
Quantum Communication

$$Z^r S^a S^b \left(S^\dagger\right)^{a\oplus b}|+\rangle \quad = \quad Z^r Z^{a\wedge b}|+\rangle$$

$$\begin{pmatrix}1 & 0\\ 0 & e^{ib\pi/2}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{ia\pi/2}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & e^{-i(a\oplus b)\pi/2}\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & e^{-i(a\wedge b)\pi}\end{pmatrix}\begin{pmatrix}1\\ -1\end{pmatrix}$$

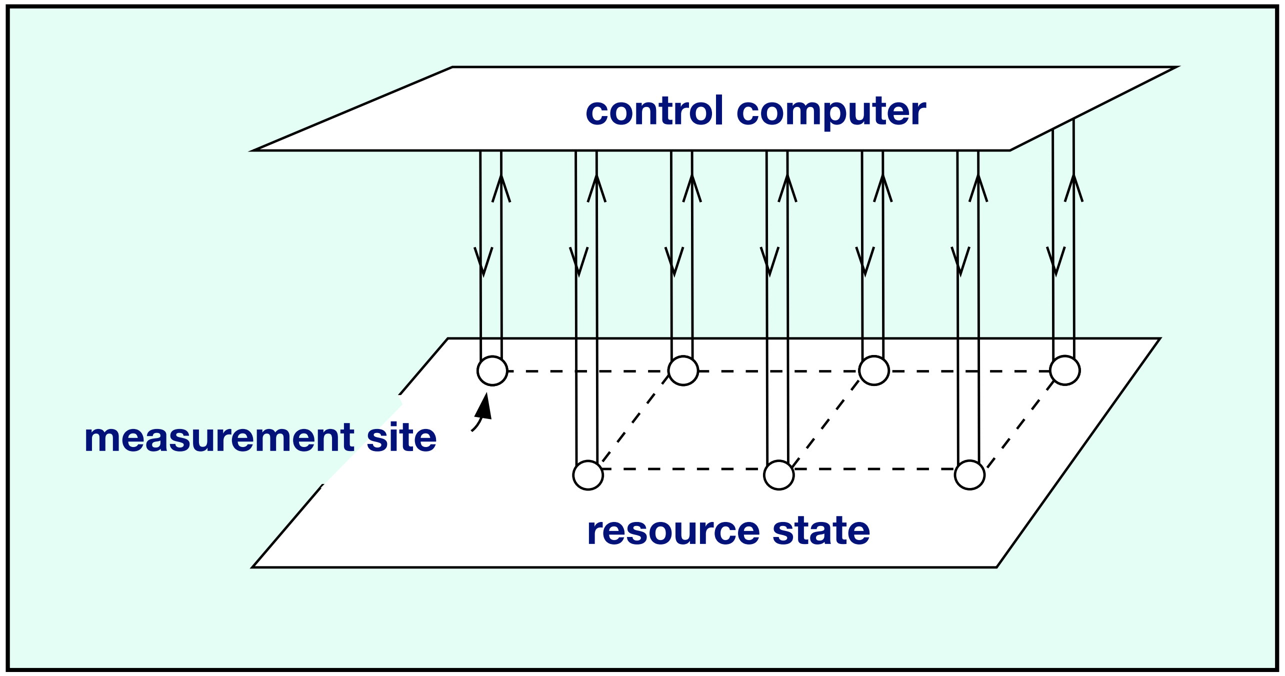
$$\begin{aligned}
\sigma_z \otimes \sigma_z \otimes \sigma_z \, |\psi\rangle &= |\psi\rangle \,, \\
\sigma_z \otimes \sigma_x \otimes \sigma_x \, |\psi\rangle &= |\psi\rangle \,, \\
\sigma_x \otimes \sigma_z \otimes \sigma_x \, |\psi\rangle &= |\psi\rangle \,, \\
\sigma_x \otimes \sigma_x \otimes \sigma_z \, |\psi\rangle &= - \, |\psi\rangle \,,
\end{aligned}$$

Secure NAND

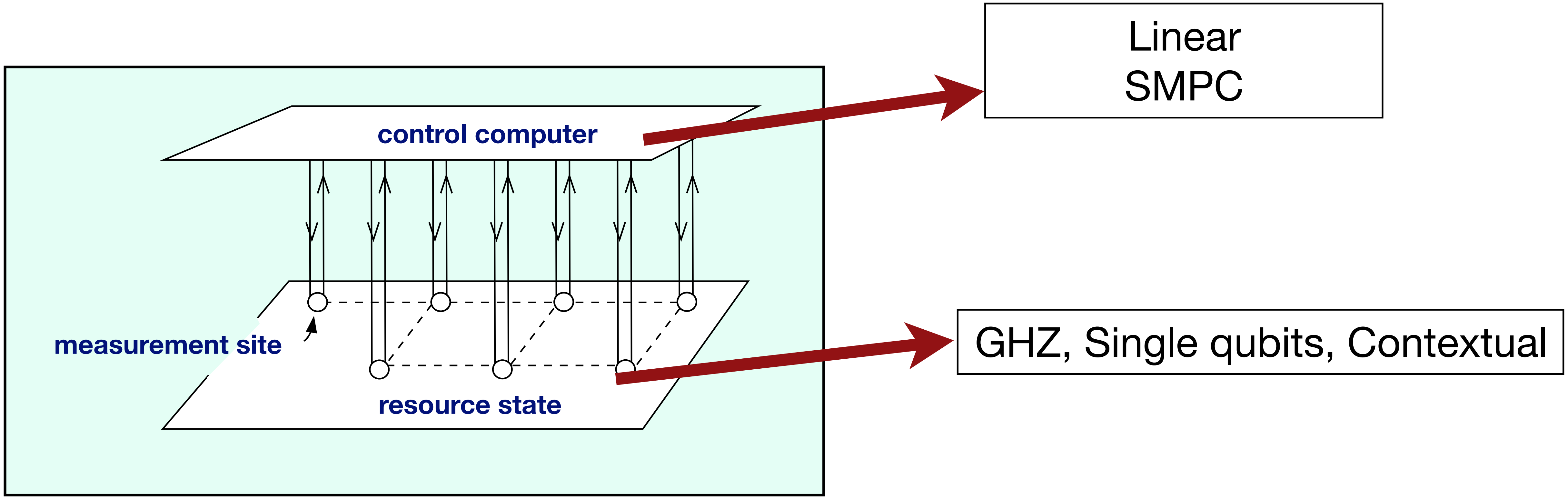


syncronization of signals necessary
50m of fiber
A
B
A XOR B
RNG
result
heralded single-photon source
coherent beam "laser pointer"
(pulsed?)
"click"
measurement device: (A) PD
Client lab
Server lab

From Linear to Non-linear - SMPC



From Linear to Non-linear - SMPC



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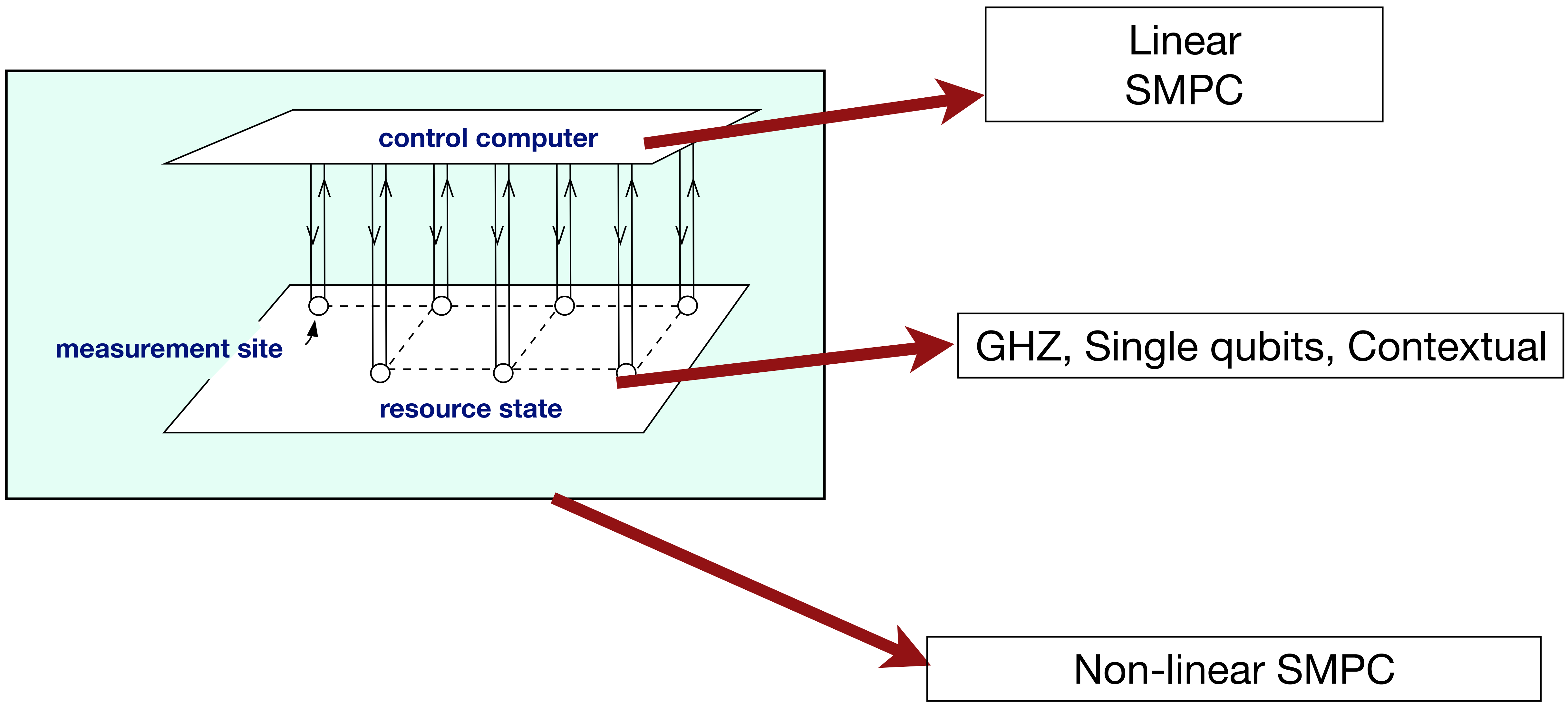

From Linear to Non-linear - SMPC

$$(S^\dagger)^{\oplus x_i} S^{x_n} \dots S^{x_1} \left|+\right\rangle = Z^{f(x_1,\dots,x_n)} \left|+\right\rangle$$

$$f(x_1,\dots,x_n) = \begin{cases} 1, \text{ if } \sum x_i = 2 \pmod 4 \text{ or } \sum x_i = 3 \pmod 4 \\ 0, \text{ if } \sum x_i = 0 \pmod 4 \text{ or } \sum x_i = 1 \pmod 4 \end{cases}$$

From Linear to Non-linear - SMPC

$$(S^\dagger)^{\oplus x_i} S^{x_n} \dots S^{x_1} \left|+\right\rangle = Z^{f(x_1,\dots,x_n)} \left|+\right\rangle$$

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$$f(x_1,\dots,x_n) = x_1 \cdot x_2 + (x_1 + x_2) \cdot x_3 + (x_1 + x_2 + x_3) \cdot x_4 + \dots + (x_1 + \dots x_{n-1}) \cdot x_n$$

From Linear to Non-linear - SMPC

$$(S^\dagger)^{\oplus x_i} S^{x_n} \dots S^{x_1} |+\rangle = Z^{f(x_1,\dots,x_n)} |+\rangle$$

$$f(x_1,\dots,x_n) = \begin{cases} 1, \text{ if } \sum x_i = 2 \pmod 4 \text{ or } \sum x_i = 3 \pmod 4 \\ 0, \text{ if } \sum x_i = 0 \pmod 4 \text{ or } \sum x_i = 1 \pmod 4 \end{cases}$$

$$f(x_1,\dots,x_n) = x_1 \cdot x_2 + (x_1 + x_2) \cdot x_3 + (x_1 + x_2 + x_3) \cdot x_4 + \dots + (x_1 + \dots x_{n-1}) \cdot x_n$$

$$\boxed{(S^\dagger)^{\oplus x_i} Z^{y_n} S^{x_n} \dots Z^{y_1} S^{x_1} |+\rangle = Z^{\oplus y_n} Z^{f(x_1,\dots,x_n)} |+\rangle}$$

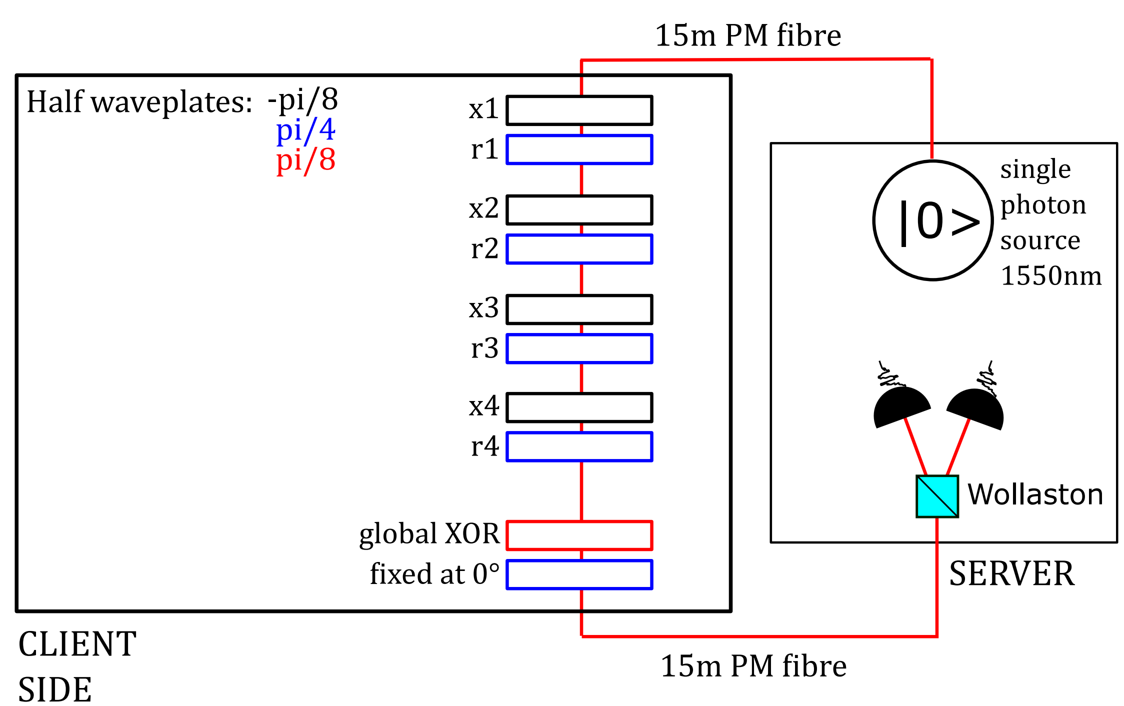
From Linear to Non-linear - SMPC

$$(S^\dagger)^{\oplus x_i} S^{x_n} \dots S^{x_1} \left|+\right\rangle = Z^{f(x_1,\dots,x_n)} \left|+\right\rangle$$

$$f(x_1,\dots,x_n) = \begin{cases} 1, \text{ if } \sum x_i = 2 \pmod 4 \text{ or } \sum x_i = 3 \pmod 4 \\ 0, \text{ if } \sum x_i = 0 \pmod 4 \text{ or } \sum x_i = 1 \pmod 4 \end{cases}$$

$$f(x_1,\dots,x_n) = x_1 \cdot x_2 + (x_1 + x_2)\cdot x_3 + (x_1 + x_2 + x_3)\cdot x_4 + \dots + (x_1 + \dots x_{n-1})\cdot x_n$$

$$\boxed{(S^\dagger)^{\oplus x_i} Z^{y_n} S^{x_n} \dots Z^{y_1} S^{x_1} \left|+\right\rangle = Z^{\oplus y_n} Z^{f(x_1,\dots,x_n)} \left|+\right\rangle}$$



From Linear to Non-linear - SMPC

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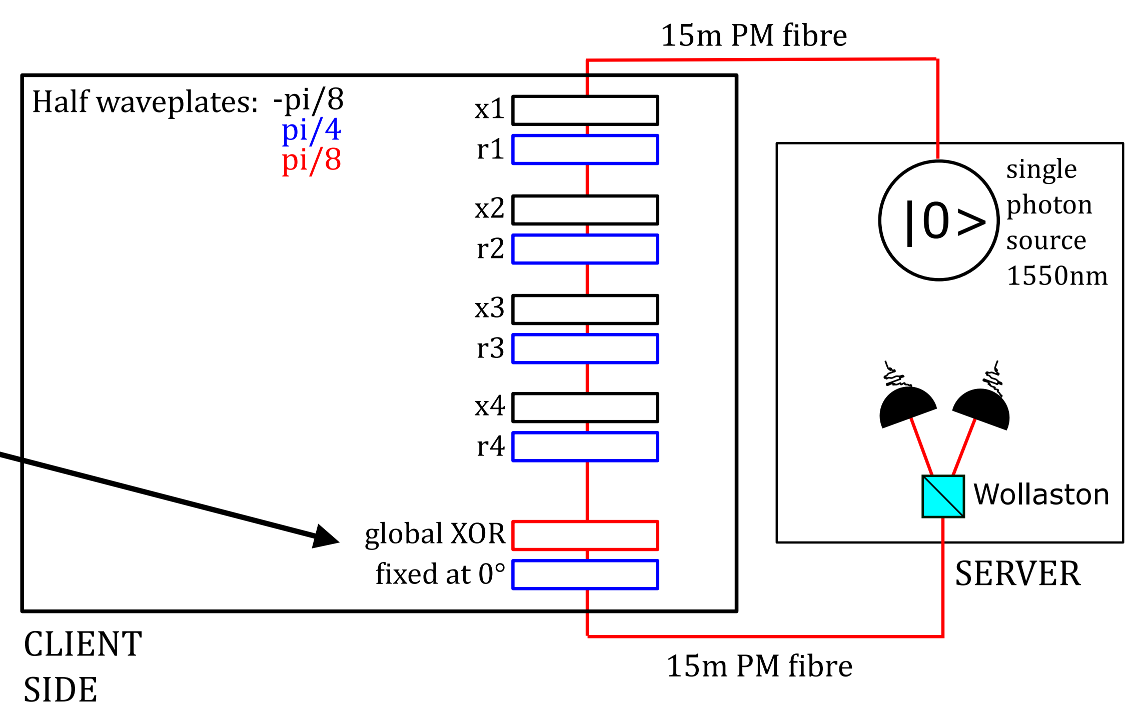
Classical Linear SMPC



Perspective

Can we do something with even few qubits ?

Classical Linear SMPC

Contextual States

Classical Non-linear SMPC

versus

Classical SMPC

Computation represented by a series of additions and multiplications of elements in F_p.

easy

Linear Verifiable Secret Sharing

hard

costly but offline FHE

Future Network

A **hybrid** network of classical protocols with quantum gadgets

boosting efficiency and security

of every task achievable against classical attackers against quantum attackers

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